Workshop on Boundary Problems and Spectral Theory
Copenhagen August 6-8

SCHEDULE

Registration is 9:30-10 Wednesday at Aud 4 close to the Department of Mathematical Sciences, University of Copenhagen. All lectures are in Auditorium 4. The workshop dinner will be at Restaurant Höst http://cofoco.dk/en/restaurants/hoest/ on Thursday 20:30.

To get internet access the easiest way is if you can use eduroam. Otherwise in the mathematics department you can use the WiFi "conference". For the password you should contact Jan Philip Solovej.

	Wednesday	Thursday	Friday
10.00–10.50	Helffer	Brown	Johnsen
11.00–11.50	Rozenblioum	Mazzeo	Wood
12:00–13.30	Lunch	Lunch	Lunch
13.30–14.20	Amann	Boutet de Monvel	Fournais
14.30–15.20	Loss	Behrndt	Malamud
15.20–15.40	Coffee	Coffee	Coffee
15.40–16.30	Laptev	Abels	Sjöstrand
16.40–17.30		Gimperlein	

Talks and Abstracts:

Helmut Abels

Title: Nonsmooth Pseudodifferential Operators and Applications

Abstract: We discuss the calculus of non-smooth pseudodifferential operators with coefficients, which have a limiting regularity with respect to the spacial variable x. Although the standard results on arbitrary compostions of pseudodifferential operators break down and the operators have limiting mapping properties, they can still be used to construct parametrizes for (parameter-)elliptic operators and various applications of it. We will discuss some applications and present a recent result on characterization of non-smooth pseudodifferential operators and spectral invariance.

Herbert Amann

Title: Parabolic equations on noncompact manifolds

Abstract: We discuss optimal well-posedness results for parabolic boundary value problems on non-compact Riemannian manifolds. We explain, in particular, the

intimate connection between manifolds with cuspidal ends and degenerate equations. As a model we investigate the Zaremba problem.

Jussi Behrndt

Title: Elliptic operators with delta-interactions on hypersurfaces

Abstract: In this talk we discuss spectral properties of self-adjoint realizations of a second-order elliptic differential expression with singular interactions of δ and δ'-type supported on a compact closed smooth hypersurface in $\mathbb{R}^n$. We pay particular attention to spectral asymptotics formulae with refined remainder estimates for the singular values of the resolvent difference between the standard self-adjoint realization and the operators with a δ and δ'-interaction, respectively. The talk is based on joint work with Gerd Grubb, Matthias Langer, and Vladimir Lotoreichik.

Louis Boutet de Monvel

Title: Residual trace of Toeplitz or pseudodifferential projectors

Abstract: We recall that the residual trace of a Toeplitz or pseudodifferential projector is always zero. This is no longer the case in general.

Malcolm Brown

Title: Scattering and inverse scattering for a left-definite SturmLiouville problem

Abstract: This work develops a scattering and an inverse scattering theory for the SturmLiouville equation $u'' + qu = \lambda wu$ where w may change sign but $q! \neq 0$. Thus the left-hand side of the equation gives rise to a positive quadratic form and one is led to a leftdefinite spectral problem. The crucial ingredient of the approach is a generalized transform built on the Jost solutions of the problem and hence termed the Jost transform and the associated Paley Wiener theorem linking growth properties of transforms with support properties of functions. One motivation for this investigation comes from the Camassa Holm equation for which the solution of the Cauchy problem can be achieved by the inverse scattering transform for $u'' + 1/4u = \lambda wu$.

Søren Fournais

Title: Counter-examples to strong diamagnetism?

Abstract: "Consider a Schrdinger operator with magnetic field $B(x)$ in 2-dimensions. The classical diamagnetic inequality implies that the ground state energy, denoted by $\lambda_1(B)$, with magnetic field is higher than the one without magnetic field. However, comparison of the ground state energies for different non-zero magnetic fields is known to be a difficult question. We consider the special case where the magnetic

field has the form $b\beta$, where b is a (large) parameter and $\beta(x)$ is a fixed function. One might hope that monotonicity for large field holds, i.e. that $\lambda_1(b_1\beta) > \lambda_1(b_2\beta)$ if $b_1 > b_2$ are sufficiently large. We will display counterexamples to this hope and discuss applications to the theory of superconductivity in the Ginzburg-Landau model. This is joint work with Mikael Persson Sundqvist.

Heiko Gimperlein

Title: Nonsmooth commutator estimates, spectral asymptotics and applications

Abstract: This talk considers the spectral asymptotics of commutators $[P, f]$ between a pseudodifferential operator P and a Hölder continuous function f. The mapping properties of such commutators have been of interest in harmonic analysis since work by Calderon in the 1960s. Much less is known about their spectral theory, which relates to the classical theory of Hankel operators and is motivated by applications.
We discuss sharp upper estimates for the asymptotic behavior of the singular values (weak-Schatten class properties) as well as explicit formulas for their Dixmier traces on closed Riemannian or sub-Riemannian manifolds. The spectral asymptotics is dominated by the points where f is not differentiable. It is not easily described in terms of pseudodifferential symbols. A mix of ideas from harmonic analysis, spectral theory and operator algebras enters into the proofs.
The results are applied to questions in noncommutative geometry and complex analysis of several variables. (joint work with M. Goffeng)

Bernard Helffer

Title: Dirichlet eigenfunctions of the square membrane: Courant's property, and A.Stern's and Pleijel's analyses .
(after Bérard- Helffer)

Abstract: The celebrated nodal domain theorem by Courant says that the number of nodal domains of an eigenfunction associated with a k-th eigenvalue of the Dirichlet Laplacian (eigenvalues listed in increasing order) should be less than or equal to k. Pleijel proved in 1956 that equality holds only for finitely many values of k. In this case we speak of the Courant sharp situation (this has a strong connection with the question of minimal spectral partitions).
If we look at the square, it is immediate that the first, second and fourth eigenvalues are Courant sharp. We would like to analyze the statement by Pleijel that there are no other cases. We also discuss some statements of Antonie Stern who was a PhD student of R. Courant and defended her PhD in 1924. Although we focus on her results concerning the square, let us mention that she also has similar results in the case of the sphere, fifty years before the paper of H. Lewy.

Jon Johnsen

Title: On parabolic final value problems

Abstract: The talk concerns the evolution problems that arise when a parabolic differential equation is considered together with conditions on the solution's terminal values and boundary values. Large classes of such problems turn out to be well posed in suitable spaces, defined by subjecting the data to certain compatibility conditions (extending this notion from work of Grubb and Solonnikov). The classical heat equation will be discussed to describe how non-trivial Dirichlt boundary data enter the theory.

Ari Laptev

Title: Spectral inequalities for non-self-adjoint Partial Differential operators

Abstract: We shall present some result on location of the spectrum of Dirac and Schrödinger operators with complex-valued potentials.

Michael Loss

Title: The Kac master equation: A review
Abstract: In 1956 Mark Kac invented what is now called the Kac master equation, describing collisions of N particles on the line. Kac's main purpose was to give a rigorous derivation of a spatially homogeneous
Boltzmann-type equation and to prove 'Approach to Equilibrium' as well. This topic, however, is not settled yet. It is known that the Kac Master equation has poor entropy production and there is some evidence that there are states which converge to equilibrium in entropy with a rate proportional to 1/N. Towards the end of the talk I present a remedy by coupling the Kac model to a thermostat and show that this improves the entropy production considerably. In fact the entropy of any state converges exponentially fast to the equilibrium with a rate that is independent of the particle number. The equations of motions for the thermostat itself can be derived from the Kac master equation for initial states in the limit in which infinitely many particles are in equilibrium at some fixed temperature.

Mark M. Malamud

Title: Perturbation determinants for elliptic boundary value problems.

Abstract: A new approach to trace formulas and perturbation determinants of a pair $\{\widetilde{A}, \widetilde{A}'\}$ of resolvent comparable operators will be discussed. Abstract results are applied to compute a perturbation determinant for a pair $\{\widetilde{A}, \widetilde{A}'\}$ of two realizations of a symmetric second order properly elliptic operator $\mathcal{A}$ in a domain Ω with compact

smooth boundary $\partial\Omega$. For any not necessarily closed operator $K : H^{-1/2}(\partial\Omega) \longrightarrow H^{-3/2}(\partial\Omega)$ we define a realization $\widehat{A}_K$ of $\mathcal{A}$ by setting

$$\widehat{A}_K := A_{\max} \upharpoonright \mathrm{dom}\,(\widehat{A}_K), \quad \mathrm{dom}\,(\widehat{A}_K) := \{f \in \mathrm{dom}\,(A_{\max}) : \gamma_1 f = K\gamma_0 f\}, \tag{1}$$

where γ_0 and γ_1 are Dirichlet and Neumann trace mappings. Denote also by A_0 the Dirichlet realization of $\mathcal{A}$, $\mathrm{dom}\,(A_0) := \{u \in H^2(\Omega) \,|\, \gamma_0 u = 0\}$, and introduce Dirichlet-to-Neumann map

$$\Lambda_s(z) : H^s(\partial\Omega) \to H^{s-1}(\partial\Omega), \quad \Lambda_s(z)\varphi = \gamma_1 P(z)\varphi, \tag{2}$$

where P is the Poisson operator and $z \in \rho(A_0)$.
Alongside the operator K we consider its restriction $K' : H^{-1/2}(\partial\Omega) \longrightarrow H^{-3/2}(\partial\Omega)$ given by

$$K' := K \upharpoonright \mathrm{dom}\,(K'), \quad \mathrm{dom}\,(K') := \{h \in \mathrm{dom}\,(K) : Kh - \Lambda_{-\frac{1}{2}}(0)h \in H^{1/2}(\partial\Omega)\}. \tag{3}$$

Clearly, $\widehat{A}_K = \widehat{A}_{K'}$.
Let $n = 2$ and let $\mathrm{dom}\,(K) \subseteq L^2(\partial\Omega)$ and $\mathrm{ran}\,(K) \subseteq L^2(\partial\Omega)$. Then

$$(K' - \Lambda_{-\frac{1}{2}}(z))^{-1} \in \mathfrak{S}_1(H^{1/2}(\partial\Omega), H^{-1/2}(\partial\Omega))$$

and one of the perturbation determinants $\Delta_{\widehat{A}_K/A_0}(\cdot)$ admits two representations

$$\Delta_{\widehat{A}_K/A_0}(z) = \det_{H^{\frac{1}{2}}}\left(I - \left(\Lambda_{-\frac{1}{2}}(z) - \Lambda_{-\frac{1}{2}}(0)\right)\left(K' - \Lambda_{-\frac{1}{2}}(0)\right)^{-1}\right) \tag{4}$$

$$= \det_{H^{-\frac{1}{2}}}\left(I - \left(K' - \Lambda_{-\frac{1}{2}}(0)\right)^{-1}\left(\Lambda_{-\frac{1}{2}}(z) - \Lambda_{-\frac{1}{2}}(0)\right)\right). \tag{5}$$

Moreover, if $\mathrm{dom}\,(K_j) \subseteq L^2(\partial\Omega)$ and $\mathrm{ran}\,(K_j) \subseteq L^2(\partial\Omega)$, $j \in \{1, 2\}$, then perturbation determinant $\Delta_{\widehat{A}_2/\widehat{A}_1}(\cdot)$ where $\widehat{A}_j := \widehat{A}_{K_j}$ is

$$\Delta_{\widehat{A}_2/\widehat{A}_1}(z) = \frac{\det_{H^{-\frac{1}{2}}}\left(I - \left(K_2' - \Lambda_{-\frac{1}{2}}(0)\right)^{-1}\left(\Lambda_{-\frac{1}{2}}(z) - \Lambda_{-\frac{1}{2}}(0)\right)\right)}{\det_{H^{-\frac{1}{2}}}\left(I - \left(K_1' - \Lambda_{-\frac{1}{2}}(0)\right)^{-1}\left(\Lambda_{-\frac{1}{2}}(z) - \Lambda_{-\frac{1}{2}}(0)\right)\right)}. \tag{6}$$

Under additional restrictions on the operators K_j the perturbation determinant $\Delta_{\widehat{A}_2/\widehat{A}_1}(\cdot)$ can be computed in the space $H^0(\partial\Omega) = L^2(\partial\Omega)$.
Note that our considerations substantially rely on classical results of G. Grubb [1] on elliptic boundary value problems. The talk is based on joint work with H. Neidhardt [2].
[1] G. Grubb, A characterization of the non-local boundary value problems associated with an elliptic operator. *Ann. Scuola Norm. Sup. Pisa (3)*, 22: 425–513, 1968.

[2] M. Malamud and H. Neidhardt, Perturbation determinants for singular perturbations. *Russ. J. Math. Phys.*, 21(1):55–98, 2014.

Rafe Mazzeo

Title: Self-adjoint boundary problems for the Hodge-de Rham operator on stratified spaces

Abstract: I will report on a series of results, obtained with Albin, Leichtnam and Piazza, and in part also Banagl, concerning the signature theorem on stratified spaces endowed with iterated edge metrics. The firstresult concerns the essential self-adjointness of the signature operator on stratifiedspaces satisfying the Witt condition, and the general higher signature theorem in this setting. Further results describe the boundaryconditions of Cheeger type that may be imposed when the Witt condition fails,and then connect the analytic signature with the purely topological signature theory of Banagl.

Grigori Rozenblioum

Some spectral properties of Toeplitz operator in Bergman spaces and related problems in real and complex analysis.

Abstract: Let D be some, bounded or unbounded, domain in the complex plane C. In the space $H = L^2(D)$ with respect to some measure we consider the subspace B^2 consisting of analytical functions; it is called the Bergman space. For a special case when $D = C$ and the measure is the Gaussian one, the space is called the Fock space. Let P be the orthogonal projection from H to B^2. For a function V on D, the Toeplitz operator T_V in B^2 is defined as $u \mapsto PVu$. Such Toeplitz operators play an important part in Mathematical Physics (for the Fock space they were introduced by F.Berezin as a part of his quantization program), real and complex analysis, spectral theory of differential operators, etc. In the talk we discuss the relation between qualitative and quantitative properties of T_V and properties of V. A special attention will be devoted to the questions on the finiteness of the nonzero spectrum and to the eigenvalue asymptotics and estimates. Some interesting questions in real and complex analysis arise in the study of these spectral properrties. The talk is based upon joint papers with A.Pushnitski, A.Borichev, A.Aleksandrov, N.Shirokov.

Johannes Sjöstrand

Title: Distribution of eigenvalues for large Jordan blocks with random perturbations.

Abstract: For large Jordan blocks with random perturbations we show that with probability very close to 1, most of the eigenvalues are close to a circle and have an approximately uniform angular distribution. We follow a method for differential

operators developed by M. Hager, W. Bordeaux Montrieux and the speaker, here simplified by the absence of microlocal analysis. Related and intersecting results by other authors will be mentioned.

Ian Wood

Title: Boundary triples and spectral information in abstract M-functions

Abstract: The Weyl-Titchmarsh m-function is an important tool in the study of forward and inverse problems for ODEs, as it contains all the spectral information of the problem. The abstract setting of boundary triples allows the introduction of an abstract operator M-function. It is then interesting to study how much spectral information is still contained in this more general setting. Boundary triples allow for the study of PDEs, block operator matrices and many other problems in one framework. We will discuss properties of M-functions, their relation to the resolvent and the spectrum of the associated operator, and connections to the extension theory of operators.

Dates for invited partipants:

- Helmut Abels (University of Regensburg) 05.08.2014 - 08.08.2014
- Herbert Amann (Univ. Zürich) 05.08.2014 - 09.08.2014
- Jussi Behrndt (Graz University of Technology) 05.08.2014 - 08.08.2014
- Anne Boutet de Monvel (Université Paris Diderot) 05.08.2014 - 08.08.2014
- Louis Boutet de Monvel (Universite Paris Jussieu) 05.08.2014 - 08.08.2014
- Malcolm Brown (University of Cardiff) 05.08.2014 - 09.08.2014
- Søren Fournais (Århus Universitet) 06.08.2014 - 08.08.2014
- Heiko Gimperlein (Heriot-Watt University) 05.08.2014 - 10.08.2014
- Bernard Helffer (Universite Paris Sud) 05.08.2014 - 09.08.2014
- Ari Laptev (Imperial College and Mittag-Leffler) 05.08.2014 - 08.08.2014
- Michael Loss (Georgia Tech) 05.08.2014-07.08.2014
- Mark Malamud (Donetsk State Univ.) 05.08.2014 - 09.08.2014
- Rafe Mazzeo (Stanford University) 05.08.2014 - 09.08.2014
- Grigori Rozenblioum (Chamlmers University, Gteborg) 05.08.2014 - 08.08.2014

- Robert Seeley (Boston University) 06.08.2014 - 09.08.2014
- Johannes Sjöstrand (Universite Bourgogne) 05.08.2014 - 09.08.2014
- Vsevolod Solonnikov (Steklov Institute St. Petersburg) 05.08.2014 - 09.08.2014
- Ian Wood (Kent University) 05.08.2014 - 09.08.2014