Quantum Minkowski Space

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Introduction

Rieffel Deformation:

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- C^*-algebras

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- $X \curvearrowleft G$ - group actions

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$$\Psi(\hat{\gamma}_1 + \hat{\gamma}_2, \hat{\gamma}_3)\Psi(\hat{\gamma}_1, \hat{\gamma}_2) = \Psi(\hat{\gamma}_1, \hat{\gamma}_2 + \hat{\gamma}_3)\Psi(\hat{\gamma}_2, \hat{\gamma}_3)$$

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- $(C_0(G)^{\bar{\Psi}\otimes\Psi}, \Delta^\Psi)$ - locally compact quantum group.

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- $\Psi(z_1, z_2) = e^{is\Im(z_1 \bar{z}_2)}$

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- Commutation relations

$$
\begin{array}{llll}
\hat{\alpha}\hat{\alpha}^* = \hat{\alpha}^*\hat{\alpha} & & & \\
\hat{\alpha}\hat{\beta}^* = t\hat{\beta}^*\hat{\alpha} & \hat{\beta}\hat{\beta}^* = \hat{\beta}^*\hat{\beta} & & \\
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\end{array}
$$

  and $\hat{\alpha}\hat{\delta} - \hat{\beta}\hat{\gamma} = 1, \ \hat{\alpha}\hat{\beta} = \hat{\beta}\hat{\alpha}, \ \hat{\alpha}\hat{\delta} = \hat{\delta}\hat{\alpha} \dots$

- $t = e^{-8s}$

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