Workshop in the honor of Kirchberg's 65th birthday
Titles and abstracts

Bruce Blackadar: *Noncommutative PL-topology.*

I will review the theory of generalized inductive limits of finite-dimensional C^*-algebras, a longstanding project with Kirchberg, and other aspects of what can be called "noncommutative PL-topology."

Nate Brown: *Dynamical systems associated to II_1 factors.*

Given II_1 factors N and M, one can consider the collection of homomorphisms from N to M, modulo unitary equivalence, and denote this by $\mathrm{Hom}(N, M)$. Obvious actions of the outer automorphism groups of N and M yield dynamical systems, and I will try to summarize what is currently known in the special case that M is an ultraproduct of the hyperfinite II_1 factor. Then I'll discuss joint work with Valerio Capraro (inspired by an idea of Ilijas Farah) which broadens the context and transforms homomorphisms into Banach spaces.

Joachim Cuntz: *Semigroups, C^*-algebras and number fields.*

With any group one can associate a natural algebra of operators on a Hilbert space. Such algebras are classically a basis for applications of the theory of operator algebras to areas such as geometry, topology or representation theory. Recently similar algebras associated with semigroups rather than groups have been studied. It turns out that, for natural semigroups arising from number fields, the associated algebra of operators has intriguing properties that reflect the structure of the field.

Marius Dadarlat: *The C^*-algebra of a vector bundle.*

We prove that the Cuntz-Pimsner algebra $\mathcal{O}_E$ of a vector bundle E of rank ≥ 2 over a compact metrizable space X is determined up to an isomorphism of $C(X)$-algebras by the ideal $(1 - [E])K^0(X)$ of the K-theory ring $K^0(X)$. Moreover, if E and F are vector bundles of rank ≥ 2, then a unital embedding of $C(X)$-algebras $\mathcal{O}_E \subset \mathcal{O}_F$ exists if and only if $1 - [E]$ is divisible by $1 - [F]$ in the ring $K^0(X)$. We introduce related, but more computable K-theory and cohomology invariants for $\mathcal{O}_E$ and study their completeness. As an application we classify the unital separable continuous fields with fibers isomorphic to the Cuntz algebra $\mathcal{O}_{m+1}$ over a finite connected CW complex X of dimension $d \leq 2m + 3$ provided that the cohomology of X has no m-torsion.

Siegfried Echterhoff: *A general Kirillov theory for certain locally compact nilpotent groups.* (Joint work with Helma Klüver.)

In this lecture we give a report on a construction, based on ideas of Howe, of a very general Kirillov theory for the computation of the primitive ideal space of the group C^*-algebra of certain locally compact nilpotent groups G. The theory works basically for all groups, to which we may associate a locally compact Lie ring $\mathfrak{g}$ together with a homeomorphism $\exp\colon \mathfrak{g} \to G$, which shares many features of the standard exponential map in Lie group theory. This class of groups covers all simply connected and connected nilpotent Lie groups, unipotent groups over $\mathbb{Q}_p$, unipotent groups over function fields with small nilpotence length, and countable divisible nilpotent groups.

Ed Effros: *Eberhard Kirchberg and the Development of Quantized Functional Analysis.*

Grothendieck was the first mathematician to realize that categorical approaches can be used to great effect in Banach space theory. This provided a key investigative tool in the operator realm, for the investigation of quantized Banach spaces (operator spaces) and their earlier order-theoretic variants, the quantized function systems (operator systems). Provided one acknowledges the matrix norms and orderings, and the corresponding notions of complete boundedness and complete positivity, one can formulate analogues of injectivity, approximation properties (e.g., nuclearity), and the realization of the Grothendieck program in the non-commutative context. Despite the dramatic successes of this theory, it soon became evident that the quantum realm entailed completely new phenomena, that simply had no classical analogues. These included the notions of the weak extension property (Lance), the existence of non-local reflexive spaces (E-Haagerup), and most dramatically the notions of exactness and the local lifting property (Kirchberg). These new directions required enormous imagination and technical ability. When Kirchberg finally obtained an audience in the late eighties, it was precisely those qualities in his work that impressed the mathematical community. When personally asked to evaluate his difficult and unedited manuscripts, I had two reactions. I couldn't follow many of his arguments, but I had a strong feeling that they were right. After sketching some of Eberhard's work, I will turn to a very surprising new interpretation of his work in the context of operator systems. Paulsen and his colleagues have shown that the theory of tensor products of operator systems leads to elegant new approaches to exactness, the WEP and the Connes embedding problem. This work will almost certainly be relevant to the recent applications to the Bell inequalities, and other aspects of quantum information theory.

George Elliott: *Reflections on non-simple inductive limits.*

Work being undertaken concerning inductive limits of circle algebras and dimension drop interval algebras will be outlined.

Uffe Haagerup: *Approximation properties for groups and von Neumann algebras.*

This talk is about recent advances concerning approximation properties for groups and group von Neumann algebras. In 1992 Jon Kraus and I introduced a new approximation property (AP) for locally compact groups and we proved that for dicrete groups AP is equivalent to the property W*-OAP of Effros and Ruan for the group von Neumann algebra. Recently Vincent Lafforgue and Michael de la Salle have proved that $\mathrm{SL}(n, \mathbb{R})$ and $\mathrm{SL}(n, \mathbb{Z})$ do not have the property AP for $n \geq 3$. In a joint work with Tim de Laat we extend their result by proving that $\mathrm{Sp}(2, \mathbb{R})$ and more generally all simple connected Lie groups of real rank ≥ 2 and with finite center do not have the AP. The proof uses some careful estimates of Jacobi polynomials obtained in collaboration with Henrik Schlichtkrull.

Ilan Hirshberg: *The decomposable approximation theorem for nuclear C^*-algebras.* (Joint work with Kirchberg and White.)

I will discuss the proof of an approximation theorem, due to Kirchberg, which states that the completely positive approximation property for nuclear C^*-algebras can be refined so as to require that the approximating maps are decomposable. That puts the concepts of decomposition rank and nuclear dimension in a broader context, as those definitions simply involve putting an upper bound on the number of summands in the decomposition. An application of this theorem to the problem of near inclusions will be discussed in the talk of Stuart White.

Paul Jolissaint: *Normalizers of group algebras and mixing.*

Let G be a discrete group and let H be a subgroup of G. Assume that G acts on some finite von Neumann algebra (Q, τ). We discuss conditions under which the normalizer algebra of the crossed product $\mathcal{N}_{Q \rtimes G}(Q \rtimes H)''$ coincides with the crossed product algebra $Q \rtimes \mathcal{N}_G(H)$. It turns out that it depends on a weakly mixing property of the pair $H < G$. We also present families of examples of such pairs of groups.

David Kerr: *Algebraic actions, entropy, and operator algebras.*

Algebraic actions (i.e., group actions on compact Abelian groups by automorphisms) provide a rich class of examples that exhibit in a prototypical and structured way various dynamical phenomena associated with the concept of entropy. As the scope of this theory has expanded over the last few years from commutative to noncommutative acting groups, operator algebras have come to play an important role. I will describe this development and discuss some recent results on the entropy of principal algebraic actions.

Eberhard Kirchberg: *A characterization of locally reflexive C^*-algebra by an "inner"version of exactness.*

We use a non-commutative version of the Egorov theorem (in measure theory) to prove that a C^*-algebra A is locally reflexive—i.e., has property (C$''$) of Effros and Haagerup—if and only if the sequence of operator spaces

$$0 \to L \otimes B \to (\mathcal{K} \otimes A) \otimes B \to ((\mathcal{K} \otimes A)/L) \otimes B \to 0$$

is exact (in the complete metric sense) for every closed left-ideal L of $\mathcal{K} \otimes A$ and every C^*-algebra B.

I will outline the ideas of the proofs, and mention some open problems concerning exactness and local reflexivity.

N. Christopher Phillips: *Towards the classification of outer actions of finite groups on purely infinite algebras.*

UCT Kirchberg algebras (purely infinite simple separable nuclear C^*-algebras satisfying the Universal Coefficient Theorem) are known to be determined up to isomorphism by K-theoretic invariants. More recently, a K-theoretic classification has been given for actions of finite groups on such algebras satisfying the Rokhlin property. We describe progress toward such a classification under the much less restrictive condition that the action be pointwise outer, with the best results being possible when the group is cyclic of prime order.

Andreas Thom: *Normal generation and ℓ^2-Betti numbers of groups.*

The *normal rank* of a group is the minimal number of elements whose normal closure coincides with the group. We study the relation between the normal rank of a group and its first ℓ^2-Betti number and conjecture that inequality $\beta_1^{(2)}(G)$ does not exceed normal rank minus 1 for torsion free groups. The conjecture is proved for limits of left-orderable amenable groups. On the other hand, for every $n \geq 2$ and every $\varepsilon > 0$, we give an example of a simple group Q (with torsion) such that $\beta_1^{(2)}(Q) \geq n-1-\varepsilon$. These groups also provide examples of simple groups of rank exactly n for every $n \geq 2$; existence of such examples for $n > 3$ was unknown until now.

Andrew Toms: *Regularity properties in nuclear C^*-algebras.*

As is well known, Kirchberg's "purely infinite iff $\mathcal{O}_\infty$-stable"result is a cornerstone of the Kirchberg-Phillips classification of what are now known simply as Kirchberg algebras. In this talk we'll discuss a possible stably finite variant of this theorem, instances where it holds, and the implications for the classification of simple separable nuclear C^*-algebras.

Simon Wassermann: *Simple non-exact C^*-algebras with no proper tensor factorisations.*

The existence of unital simple C^*-algebras which are not isomorphic to a tensor product of two infinite dimensional C^*-algebras will be considered. By results of Ozawa, there exist separable exact examples of algebras with this property. We shall give results on the existence of uncountably many mutually non-isomorphic separable non-exact examples.

Stuart White: *Linearising near inclusions.*

This is a continuation of Ilan Hirshberg's talk reporting on joint work with Eberhard Kirchberg and Ilan Hirshberg. I'll show how to use the decomposition theorem Ilan will present to linearise certain near inclusions and discuss how this can be used to obtain a C^*-version of a von Neumann embedding result of Christensen from 1980.

Wilhelm Winter: *Dimension and regularity of nuclear C^*-algebras.*

I will give a survey of recent developments in the structure theory of nuclear C^*-algebras, on which Eberhard Kirchberg's work had a great influence. I will then describe how some of these ideas are reflected in topological dynamics.