Design and Development and the Systematic Improvement of Practice

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Outline

Brief history and framework

Tasks in mathematics education

Design

• principles, tactics and technique for tools for
  • curriculum, assessment, professional development

Professional development tools

Strategic and structural design issues

Why is large-scale improvement so difficult?
To work to improve the teaching and learning of mathematics regionally, nationally and internationally.

For us this implied:

- Focus on direct **impact** on practice in classrooms
- Scale can only be achieved through **materials**
- **Engineering style research** >> products + insights
- Focus on **design**: strategic, structural, technical
Result: A sequence of linked R&D projects

Developing tools and processes for

- Classroom teaching and learning
- Assessment – formative and summative
- Teacher professional development
- Systemic change

Key principles

- Input from prior *insight* research, ours and others’
- Imaginative design
- Systematic development through observed trials in realistic conditions
Some Shell Centre projects

Testing Strategic Skills 1980-88
• Exam-driven gradual change, integrated support, “tests worth teaching to”

Diagnostic Teaching 1984-93
• Learning through misconceptions, cognitive conflict

Investigations on Teaching with Microcomputers as an Aid 1980-88
• Potential of a “computer whiteboard” to stimulate investigation

Balanced Assessment/MARS 1992-2010
• US, Classroom assessment, Framework for balance, Exponential ramp

World Class Arena 1999-2005
• Test of PS across STEM, Expert computer-based tasks, Teaching support

Improving Learning in Mathematics 2004-5
• multimedia PD on developing conceptual understanding for schools and colleges

Bowland Maths 2006-10
• Investigative video/software driven microworlds, PD packages

Mathematics Assessment Project 2009-15
• Supporting formative assessment (~ diagnostic teaching) through materials; tasks
Projects and products
Examples from Mathematics Assessment Project

map.mathshell.org

The Mathematics Assessment Project

“And I’m calling on our nation’s governors and state education chiefs to develop standards and assessments that don’t simply measure whether students can fill in a bubble on a test, but whether they possess 21st Century skills like problem solving and critical thinking and entrepreneurship and creativity.”

President Obama, 1 March 2009.

Project goals

The project is working to design and develop well-engineered assessment tools to support US schools in implementing the Common Core State Standards for Mathematics (CCSSM).

Products

Tools for formative and summative assessment that make knowledge and reasoning visible, and help teachers to guide students in how to improve, and monitor their progress. These tools comprise:

- **Classroom Challenges**: lessons for formative assessment, some focused on developing math concepts, others on non-routine problem solving.
- **Professional Development Modules**: to help teachers with the new pedagogical challenges that formative assessment presents.
- **Summative Assessment Task Collection**: to illustrate the range of performance goals required by CCSSM.
- **Prototype Summative Tests**: designed to help teachers and students monitor their progress, these tests provide a model for examinations that may replace or complement current US tests.

The team also contributes to some system capacity building activities within the wider collaboration that the Gates Foundation has assembled, including states and school systems across the US.

The Team

The project is a collaboration between the Shell Center team at the University of Nottingham and the University of
Our Design Team

Malcolm Swan

Hugh Burkhardt

Geoff Wake

Colin Foster

Rita Crust

Daniel Pead

Clare Dawson

Sheila Evans
Malcolm Swan - lead designer
Tasks in mathematics education
What are tasks for?

Assessing students’ performance via
  • tests, coursework, formative assessment

Providing ‘microworlds’ for learning
  • lessons built around tasks, preferably rich tasks

but also

Providing targets for performance
  • ‘past exam papers’ > Fermat’s last theorem, Hilbert problems, travelling salesman problem
    >> “ladder of problem solving tasks” map.mathshell.org

Summarising curriculum goals
  • complementing domain descriptions
Task Difficulty

Difficulty depends on:

- Complexity, Unfamiliarity, Technical demand, Reasoning time expected of the student

We have found it useful to distinguish:

- **Expert Tasks**
  the form they naturally arise in, involve all four aspects, so must not be technically demanding – “the few year gap”

- **Apprentice Tasks**
  expert tasks with scaffolding added, reduces complexity and student autonomy

- **Novice Tasks**
  short items with mainly technical demand, so can be “up to grade”, including concepts and skills just learnt

*Each has a different balance of sources of difficulty*
Airplane turn-round

How quickly could they do it?

<table>
<thead>
<tr>
<th>Job</th>
<th>Time needed</th>
</tr>
</thead>
<tbody>
<tr>
<td>A  Get passengers out of the cabin and off the plane</td>
<td>10 minutes</td>
</tr>
<tr>
<td>B  Clean the cabin</td>
<td>20 minutes</td>
</tr>
<tr>
<td>C  Refuel the plane</td>
<td>40 minutes</td>
</tr>
<tr>
<td>D  Unload the baggage from the cargo hold</td>
<td>25 minutes</td>
</tr>
<tr>
<td>E  Get new passengers on the plane</td>
<td>25 minutes</td>
</tr>
<tr>
<td>F  Load the baggage into the cargo hold</td>
<td>35 minutes</td>
</tr>
<tr>
<td>G  Do a final safety check before lift-off</td>
<td>5 minutes</td>
</tr>
</tbody>
</table>
Traffic Jam

1. Queue is 12 miles long on a two-lane freeway.
   How many cars are in the traffic jam?

2. Drivers have a two-second reaction time.
   When the accident clears, how long before the last car moves?
Table Tiles

Maria makes square tables, then sticks tiles to the top.

Maria uses whole tiles in the middle, quarter tiles at the corners and half tiles along the edges.

Describe a quick method for calculating the number of tiles of each type that are needed for any square table top.
Task Difficulty: Expert Tasks

Expert Tasks
tasks in the form they naturally arise

Difficulty comes mainly from:

- **Complexity** with various factors, not all stated
- **Unfamiliarity** so you have to work out what to do
- Technical demand
- **Autonomous reasoning** – you have to construct a chain

so they must not be technically demanding – “the few year gap”
An “Apprentice” Task

Skeleton Tower

How many cubes do you need to make a tower

• 6 cubes high?
• 20 cubes high?
• \( n \) cubes high?
• Explain your reasoning.
• Can you find another method?
“Novice” tasks

1. \( 14 \times 32 = \)

2. Write \( 3 \times 10^5 \) as an ordinary number

3. Factorise \( x^2 + 3x - 4 \)

4. Solve \( 3x + 5 = 21 - 5x \)

5. Write \( \sin(A + B) \) in terms of the \( \sin \) and \( \cos \) of \( A \) and \( B \)
A Novice Task

These three graphs show the functions:

\[ y = x^2 \]
\[ y = x^2 + k \]
\[ y = k \, x^2 \]

Where: \( k > 1 \)

Label the three graphs
A “novice” task

F-LE

29. One of these tables represents a linear relationship, one represents an exponential growth and one represents an exponential decay. Label each table correctly.

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td>15</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
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<tbody>
<tr>
<td>1</td>
<td>56</td>
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<tr>
<td>2</td>
<td>28</td>
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<td>3</td>
<td>14</td>
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<td>4</td>
<td>7</td>
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</table>

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
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<tr>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
</tr>
<tr>
<td>3</td>
<td>13.5</td>
</tr>
<tr>
<td>4</td>
<td>20.25</td>
</tr>
</tbody>
</table>
Novice Tasks
short items, “up to grade” i.e on concepts and skills just learnt

Difficulty mainly from:
• Complexity
• Unfamiliarity
• Technical demand
• Autonomous reasoning expected of the student

If also complex or unfamiliar, they will be too difficult
The Need

A world class mathematics education needs substantial experience of all three kinds of task in curriculum and assessment:

- **Novice tasks**
  tools of the trade

- **Apprentice tasks**
  guided route to expertise

- **Expert tasks**
  long chains of student reasoning + mathematics beyond the classroom – **the ultimate goal**

**Currently, many countries have only Novice Math Ed**

Key symptom: short tasks, short chains of reasoning
Projects and Products

a quick sampling
Shell Centre Projects: some design features

ITMA 1978-88
“Investigations on Teaching with Microcomputers as an Aid”
  • Microworlds, role-shifting, systematic methodology

Testing Strategic Skills 1981-86
  • WYTIWYG, Gradual change, Boxes, Alignment

Numeracy through Problem Solving 1978-88
  • Modelling in maths, materials-directed project work, group investigations, exams on projects, controlled transfer distance

Extended Tasks for GCSE Maths 1985-88
  • Materials supporting ‘coursework’/portfolios
ITMA: microworlds
Role Shifting

Directive roles

• Manager
• Explainer
• Task setter
Role Shifting

Directive roles

- Manager  \( T \rightarrow S \)
- Explainer  \( T \rightarrow S \)
- Task setter  \( T \rightarrow S \)

Supportive roles

- Counsellor  \( T \rightarrow S \)
- Fellow student  \( T \)
- Resource  \( T \)

Role shifting raises levels of learning – changes the “classroom contract”
A variety of tasks – and student roles

Plan and organise
- Find an optimum solution subject to constraints.

Design and make
- Design an artefact or procedure and test it

Model and explain
- Invent, explain models, make reasoned estimates

Explore and discover
- Find relationships, make predictions

Interpret and translate
- Deduce information, translate representations

Evaluate and improve
- An argument, a plan, an artefact
Testing Strategic Skills: Hurdles Race

The rough sketch graph shown above shows what happened when three athletes A, B and C entered a 400 metres hurdles race.

Imagine you are the race commentator. Describe what is happening as carefully as you can. You do not need to measure anything accurately.
Bowland Maths: Reducing road accidents
Bowland Maths: Reducing road accidents

![Graph showing number of casualties vs. Age]
Bowland Maths: Reducing road accidents

<table>
<thead>
<tr>
<th>Vehicle</th>
<th>Casualties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pedestrian</td>
<td>54</td>
</tr>
<tr>
<td>Cycle</td>
<td>18</td>
</tr>
<tr>
<td>Motorbike</td>
<td>12</td>
</tr>
<tr>
<td>Car</td>
<td>36</td>
</tr>
</tbody>
</table>
Preparing a case

Each group of students is allocated a budget of £100,000 to spend on road improvements.

| Road Safety campaign | A poster and leaflet campaign can be effective when it targets a particular cause of accidents. You will need to describe
|                      | • the focus of the campaign,
|                      | • the time of year it will appear,
|                      | • the type of person it will target. You need to renew the campaign each year for it to continue having an effect. |
|                      | £20,000 per year |
| Traffic lights       | Traffic lights can control the flow of traffic at junctions or other hazards, stopping some traffic while other traffic is allowed to go. |
|                      | £30,000 per junction |
| Mini roundabout      | Mini-roundabouts are often only marked out with white paint. They are used on roads that have an average speed of 30mph or less. They are often used to reduce speed before a series of road humps. |
|                      | £10,000 |
| Large roundabout     | Large roundabouts are used to control the flow of traffic at junctions between major roads. |
|                      | £40,500 |
| Road narrowings      | Road narrowings slow traffic down by forcing one stream to give-way to the other. When they are on both sides of the road they are called chicanes or pinch points. |
|                      | £10,000 |
| Pelican crossing     | Pelican crossings control vehicle and pedestrian movements with traffic lights. Pedestrians must wait for the 'green man' before crossing the road. |
|                      | £18,000 |
| Cycle lane           | Cycle lanes help keep bikes separate from other road users. They can be either on the side of the road or off-road. |
|                      | £60 per metre |
| Traffic island and pedestrian refuge | Traffic islands in the centre of a road to help reduce vehicle speeds and stop overtaking. If it includes a gap in the middle of the island it is called a refuge; it allows pedestrians to cross half the road at a time. |
|                      | £3,000 |
| Speed camera         | Speed cameras automatically photograph the number plates of drivers exceeding the speed limit. Many speeding drivers have been convicted by the photographic evidence. |
|                      | £25,000 |
| Speed humps          | Road humps can only be put on roads with speed limits of 30 mph or less. A series of humps should be about 50 metres apart and have a speed reducing feature at both ends, such as a road narrowing or mini roundabout. |
|                      | £1,000 per hump |
| School crossing patrol | A lollipop lady can help to ensure the safety of younger children. It is helpful if approaching traffic is slowed down by other measures. |
|                      | £5,000 per year |
A sample of students’ work

The problems we have found are that outside the school there are a number of accidents to cyclist leaving school.

To sort out this problem we are going to put in 500 metres of cycle lanes and tracks costing £30,000 out of our £100,000.

We could also add 8 road humps costing £8,000 To help the road humps we would need 2 traffic island and pedestrians refuge costing £6,000.

We should get 2 sets of pelican crossing costing £36,000 coming to a total cost of £80,000 this should reduce the accident rate by half. Hopefully helping save pupils lives on bikes every year.
Designing for Learning
Design Foci

**Technical design**

research input, creative design, systematic development

- detail leads to learning with “surprise and delight” in
- supporting problem solving, concept debugging, adaptive teaching, technology

**Structural design**

tool/process features that fit both ‘job’ and ‘user’ well

- support and liberate users, align all elements,..
- users are typical teachers in real classrooms, PD leaders, ...

**Strategic design**

looks for a good fit to the system: “points of leverage”

- change models, guiding policy, eg assessment
We design tasks with various learning priorities

Technical fluency
  • in recalling facts and performing skills

Conceptual understanding
  • and interpretations for representations

Strategies
  • for investigation, modelling, problem solving

Appreciation
  • of the power of mathematics in society
An analogy: ‘own language’ teaching

Develop technical accuracy

• spelling, grammar, punctuation.

Creating **texts** in different genres

• reports, letters, stories, poems, speeches.

Appreciating **texts** produced by others

• interpreting novels or plays;
• analysing dramatic techniques, structures;
• relating them to social, historical and cultural contexts.

For mathematics, just change “texts” to “tasks”
Principles from theories of learning

Students learn through

- **active processing** – discussion, reflection, social classroom
- **internalisation and reorganisation** of experience.

Activate **pre-existing concepts**.

Allow students to build **multiple connections**.

Stimulate **tension / conflict** to promote re-interpretation, reformulation and accommodation.

**Devolve problems** to students.

Students need to **articulate Interpretations**.

‘Production of answers’ must give way to **reflective periods of ‘stillness’** for examining alternative meanings and methods.

**Reasoning** – not just answers
TRU: Teaching for Robust Understanding – the five dimensions of powerful classrooms

Alan Schoenfeld with MAP and ATS teams

The Content
• Is the mathematics worthwhile – deep and connected?

Cognitive Demand
• Are the students engaged in productive struggle?

Equitable Access to Content
• Does everyone engage with the maths – or can they hide?

Agency, Ownership, and Identity
• Whose maths is it? Do students explain their ideas? Are these recognized and built on?

Formative Assessment
• Does instruction respond to the discussions and help students think more deeply?
Observe the Lesson Through a Student’s Eyes

The Content
• What’s the big idea in this lesson?
• How does it connect to what I already know?

Cognitive Demand
• How long am I given to think, and to make sense of things?
• What happens when I get stuck?
• Am I invited to explain things, or just give answers?

Equitable Access to Content
• Do I get to participate in meaningful math learning?
• Can I hide or be ignored? In what ways am I kept engaged?

Agency, Ownership, and Identity
• What opportunities do I have to explain my ideas? Are they built on?
• How am I recognized as being capable and able to contribute?

Formative Assessment
• How is my thinking included in classroom discussions?
• Does instruction respond to my ideas and help me think more deeply?
Lesson Design for Formative Assessment
Formative assessment is
Students and teachers
Using evidence of learning
To adapt teaching and learning
To meet immediate needs
Minute-to-minute and day-by-day

(Thompson and Wiliam, 2007)

MAP: 100 Classroom Challenges:
Formative assessment lessons
for US Grades 6 through 11

Over 5,000,000 lesson downloads so far
Formative assessment can link TRU dimensions
Different purposes result in different priorities

- Concept focused lessons
  - Mathematical topic
  - Applications

- Problem solving focused lessons
  - Problem
  - Choose appropriate mathematical tools
Developing Conceptual Understanding
MAP: Structure of a concept development lesson

Expose and explore students’ existing ideas
  • “pull back the rug”

Confront with implications, contradictions, obstacles
  • provoke ‘tension’ and ‘cognitive conflict’

Resolve conflict through discussion
  • allow time for formulation of new concepts.

Generalise, extend and link learning
  • apply to new contexts.
‘Diagnostic Teaching’ Research

Reflections

Rates

Decimals

- Conflict discussion (n = 26)
- Guided discovery (n = 29)

- Conflict Discussion (n = 123)
- Expository (n = 34)

- Conflict (n = 22)
- Expository (n = 25)
Formative assessment v Direct instruction
Task genres for concepts

Interpreting and translating representations
  • what is another way of showing this?

Classifying, naming and defining objects
  • what is the same and what is different?

Testing assertions and misconceptions
  • always, sometimes or never true?

Modifying problems. Exploring structure
  • what happens if I change this?
  • How will it affect this?
A sheet of questions *Percent changes* is given to students for homework before the lesson, or in the previous lesson, including:

In a sale, all prices in a shop were decreased by 20%.

After the sale they were all increased by 20%.

What was the overall effect on the shop prices?

*Explain how you know.*
A common misconception here is:

*Price - 20% + 20% = Price*

*giving no overall change – “You just add % changes”*

*Real understanding involves knowing that we are combining multipliers:*

*Price x 0.8 x 1.20 = Price x 0.96*

* - a 4% reduction

This lesson is designed to enable students detect and correct their own and each others misconceptions – and build connections (Re-teaching doesn’t work!)
Collaborative activity

“Today, I want you to work in groups of two or three. I will give each group a set of cards.”

“There is a lot of work to do today, and it doesn't matter if you don't all finish. The important thing is to learn something new, so take your time.”

“I want you to work as a team. Take it in turns to place the cards on the table and explain all your reasoning to your partner.”
### Common issues:

<table>
<thead>
<tr>
<th>Common issues</th>
<th>Suggested questions and prompts</th>
</tr>
</thead>
</table>
| Makes the incorrect assumption that a percent increase/decrease means the calculation must include an addition/subtraction | • Does your answer make sense? Can you check that it is correct?  
• “Compared to last year 50% more people attended the festival.” What does this mean? Describe in words how you can work out how many people attended the festival this year. Give me an example.  
• In a sale an item is marked “50% off.” What does this mean? Describe in words how you calculate the price of an item in the sale. Give me an example.  
• Can you express the increase/decrease as a single multiplication? |
| For example: 40.85 + 0.6 or 40.85 + 1.6 (Q1).                               |                                 |
| A single multiplication by 1.06 is enough.                                  |                                 |
| Or: 56.99 – 0.45 or 56.99 – 1.45 (Q2).                                      |                                 |
| A single multiplication by 0.55 is enough.                                 |                                 |
| Converts the percent to a decimal incorrectly                              | • How can you write 50% as a decimal?  
• How can you write 5% as a decimal? |
Plenary discussion

Conclude the lesson by discussing and generalising what has been learned.

The generalisation can be done by first extending what has been learned to new examples:

*If prices increase by 10%...*

*How can I say that as a decimal multiplication?*

*How can I write that as a fraction multiplication?*

*How much will prices need to go down to get back to the original price?*

*How can you write that as a decimal multiplication?*

*How can you write that as a fraction multiplication?*
Connections v Fragmentation

Learning involves active processing, linking new inputs to existing cognitive structure (J. Bruner and others)

Teaching math incrementally makes this harder

Novice tasks alone mean fragmentation

Design goal: to help students understand results from different perspectives

• ”If you find a result one way, it is worth thinking about
• If you show it in two ways, it may well be true
• If you can show it three ways, it probably is.”

Richard Feynman
Proportion – four ways

In the grocery store,
4 lb of tomatoes costs $5.
How much will 7 lb cost?

Abdul, Dorothy, Stef and Tim work out the answer in four different ways.
Abdul explains his method, but the others don’t.

Write in explanations, and units, that justify their work
The ratio of the cost must be the same as the ratio of the weights.

So

\[ \frac{\text{Cost}}{\$5} = \frac{7 \text{ lb}}{4 \text{ lb}} \]

Multiplying both sides by $5,

\[ \text{Cost} = \$5 \times \frac{7}{4} \]

\[ \text{Cost} = \$8.75 \]

\[ \frac{5}{4} = 1.25 \]

\[ \text{Cost} = 7 \times 1.25 = \$8.75 \]
Stef

\[ \frac{7}{4} \]

\[ \text{Cost} = \$5 \times \frac{7}{4} \]

\[ = \$8.75 \]

Tim

\[ \frac{\text{Cost}}{7} = \frac{5}{4} \]

So \[ \text{Cost} = 7 \times \frac{5}{4} \]

\[ = \$8.75 \]
Task genres for concepts

Interpreting and translating representations
  • what is another way of showing this?

Classifying, naming and defining objects
  • what is the same and what is different?

Testing assertions and misconceptions
  • always, sometimes or never true?

Modifying problems. Exploring structure
  • what happens if I change this?
  • How will it affect this?
Always, sometimes or never true?

When you cut a piece off a shape you reduce its area and perimeter.
Always, sometimes or never true?

\[
\frac{a + b}{2} \geq \sqrt{ab}
\]
### Always, Sometimes or Never True?

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$x - 6 = 6 - x$</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>$\frac{x}{6} = \frac{6}{x}$</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>$2(x - 3) = 2x - 3$</td>
<td>6</td>
</tr>
<tr>
<td>7</td>
<td>$\frac{x + 6}{2} = x + 3$</td>
<td>8</td>
</tr>
<tr>
<td>9</td>
<td>$(x + 3)^2 = x^2 + 3^2$</td>
<td>10</td>
</tr>
<tr>
<td>11</td>
<td>$(3x)^2 = 9x^2$</td>
<td>12</td>
</tr>
<tr>
<td>13</td>
<td>$x^2 + 6 = 0$</td>
<td>14</td>
</tr>
</tbody>
</table>
Task genres for concepts

Interpreting and translating representations
  • what is another way of showing this?

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Modifying problems. Exploring structure
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Making and selling candles

Teresa has bought a kit for making candles. It cost $50.

It contains enough wax and wicks to make 60 candles.

Teresa plans to sell the candles for $4 each.

If she sells them all, how much profit will she make?

\[ p = 60 \times 4 - 50 \]
Making and selling candles

The cost of buying the kit (includes molds, wax, wicks) $50

The number of candles that can be made with the kit 60 candles

The price at which she sells each candle $4

Total profit made if all candles are sold. $190

\[
p = 60 \times 4 - 50 \quad \quad p = ns - k
\]
Making and selling candles

<table>
<thead>
<tr>
<th>The cost of buying the kit (includes molds, wax, wicks)</th>
<th>$50</th>
</tr>
</thead>
<tbody>
<tr>
<td>The number of candles that can be made with the kit</td>
<td>60</td>
</tr>
<tr>
<td>The price at which she sells each candle</td>
<td>?</td>
</tr>
<tr>
<td>Total profit made if all candles are sold.</td>
<td>190</td>
</tr>
</tbody>
</table>

\[ S = \frac{190 + 50}{60} \quad S = \frac{p + k}{n} \]
Making and selling candles

The cost of buying the kit (includes molds, wax, wicks) $50

The number of candles that can be made with the kit candles

The price at which she sells each candle $4

Total profit made if all candles are sold.

\[ p = 4n - 50 \]

<table>
<thead>
<tr>
<th>n</th>
<th>0</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>p</td>
<td>-50</td>
<td>-10</td>
<td>30</td>
<td>70</td>
<td>110</td>
<td>150</td>
</tr>
</tbody>
</table>
## Making and selling candles

<table>
<thead>
<tr>
<th>Description</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>The cost of buying the kit (includes molds, wax, wicks)</td>
<td>$k$</td>
<td>$____$</td>
</tr>
<tr>
<td>The number of candles that can be made with the kit</td>
<td>$n$</td>
<td>$______$ candles</td>
</tr>
<tr>
<td>The price at which she sells each candle</td>
<td>$s$</td>
<td>$______$</td>
</tr>
<tr>
<td>Total profit made if all candles are sold</td>
<td>$p$</td>
<td>$______$</td>
</tr>
</tbody>
</table>

$$p = ns - k \quad s = \frac{p + k}{n} \quad n = \frac{p + k}{s} \quad k = ns - p$$
Developing strategies for problem solving
A sequence of problem solving materials
Problem solving

“A problem is a task that the individual wants to achieve, and for which he or she does not have access to a straightforward means of solution.”

(Schoenfeld, 1985)

“.... problems should relate both to the application of mathematics to everyday situations within the pupils' experience, and also to situations which are unfamiliar. For many pupils this will require a great deal of discussion and oral work before even very simple problems can be tackled in written form.”

(UK Cockcroft Report, 1982, para 249)
The Processes of Modelling

The real world

Mathematics

Situation → Represent Formulate → Analyse Solve → Interpret

Validate

Report
MAP: Structure of a Problem Solving Lesson

• **Initial, individual, unscaffolded problem**
  – Students tackle the problem unaided.
    Teacher assesses work and prepares qualitative feedback.

• **Individual work**
  – Students write responses to teacher’s feedback

• **Collaborative work**
  – Students work together to produce and share joint solutions

• **Students compare different approaches using sample work**
  – Students discuss student work in small groups, then as a whole class

**Whole class discussion: the payoff of mathematics**

• Students improve their solutions to the initial problem, or one very much like it.

• **Individual reflection**
  – Students write about what they have learned.
Boomerangs

Phil and Cath make and sell boomerangs for a school event. They plan to make them in **two sizes: small and large**. Phil will carve them from wood.

The **small** boomerang takes **2 hours** to carve and the **large** one takes **3 hours**. Phil has a **total of 24 hours** for carving.

Cath will decorate them. She only has time to decorate **10 boomerangs of either size**.

The **small** boomerang will make **$8** for charity. The **large** boomerang will make **$10** for charity. They want to make as much money as they can.

**How many small and large boomerangs should they make?**
**How much money will they then make?**
Phil can only make 12 small or 8 large boomerangs in 24 hours.

12 small makes $96
8 large makes $80

Cath only has time to make 10, so $96 is impossible.
She could make 10 small boomerangs which will make $80.
So she either makes 8 large or 10 small boomerangs and makes $80.

<table>
<thead>
<tr>
<th>No of Small s</th>
<th>5×8</th>
<th>No of large</th>
<th>1×10</th>
<th>Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>8</td>
<td>80</td>
<td>80</td>
</tr>
<tr>
<td>1</td>
<td>8</td>
<td>7</td>
<td>70</td>
<td>78</td>
</tr>
<tr>
<td>2</td>
<td>16</td>
<td>6</td>
<td>60</td>
<td>76</td>
</tr>
<tr>
<td>3</td>
<td>24</td>
<td>5</td>
<td>50</td>
<td>74</td>
</tr>
<tr>
<td>4</td>
<td>32</td>
<td>5</td>
<td>50</td>
<td>82</td>
</tr>
<tr>
<td>5</td>
<td>40</td>
<td>4</td>
<td>40</td>
<td>80</td>
</tr>
<tr>
<td>6</td>
<td>48</td>
<td>3</td>
<td>30</td>
<td>78</td>
</tr>
</tbody>
</table>

The most profit is $82.

Small boomerangs = x
Large boomerangs = y

Time to carve 2x + 3y = 24  (1)
Only 10 can be decorated  x + y = 10   (2)
2x + 2y = 20   (3)
10 - 3  y = 4  x = 6

So make 4 large boomerangs
6 small boomerangs.
Model and Explain

Cats and Kittens

Cats can't add but they can multiply!

In just 18 months, this female cat can have 2000 descendants

Make sure your cat cannot have kittens

Length of pregnancy
- About 2 months

Number of kittens in a litter
- Usually 4 to 6

Average number of litters a female cat can have in one year
- About 3

Age at which a female cat can first get pregnant
- About 4 months

Age at which a female cat no longer has kittens
- About 10 years

Is this figure of 2000 realistic?
Sample student work

A cat could have 24 kittens 2000 is not realistic.
Sample student work

Total cats = $1 + 6 \times 6 + 6 \times 36$

= $1 + 36 + 216$

= 253

So it's not realistic
(3 litters = 18 kittens.)

(mummy is 19)

1st litter will be born in:

the 1st litter will be born in April

2nd litter will be born in:

8th

will be able to have babies in April/born in June

2nd

will be able to have babies in August, born in November

3rd

will be able to have babies in March/May

First babys in a year yay!

9846

Conclusion.

The mother has

18 kittens in a

year each litter

are 6 kittens in each.

In a year and a half the most the family will have a
## Common issues tables

<table>
<thead>
<tr>
<th>Issue</th>
<th>Question</th>
</tr>
</thead>
<tbody>
<tr>
<td>Has difficulty starting</td>
<td>Can you describe what happens during first five months?</td>
</tr>
<tr>
<td>Does not develop suitable representation</td>
<td>Can you make a diagram or table to show what is happening?</td>
</tr>
<tr>
<td>Work is unsystematic</td>
<td>Could you start by just looking at the litters from the first cat? What would you do after that?</td>
</tr>
<tr>
<td>Develops a partial model</td>
<td>Do you think the first litter of kittens will have time to grow and have litters of their own? What about their kittens?</td>
</tr>
<tr>
<td>Does not make clear or reasonable assumptions</td>
<td>What assumptions have you made? Are all your kittens are born at the beginning of the year?</td>
</tr>
<tr>
<td>Makes a successful attempt</td>
<td>How could you check this using a different method?</td>
</tr>
</tbody>
</table>
Medical Testing

A new medical test has been invented to help doctors find out whether or not someone has got a deadly disease. Experiments have shown that:

• If a person has the disease, the test result will always be positive.
• If a person does not have the disease, then the probability that the test is wrong is 5%. This is called a **false positive** result.

| Sample Size: 15 | Healthy: 13 | False Positive: 1 |
Medical Testing

The test is tried out in two different countries: Country A and Country B.

A sample of one thousand people is tested from each country.
• In Country A, 20% of the sample has the disease.
• In Country B, 2% of the sample has the disease.

A patient from each sample is told that they have tested positive. What is the probability that the test is wrong? Is your answer the same for each country? Explain.
200 people have the disease, 800 don’t.
No of people with a false positive = 5% of 800 = 40
Probability of a wrong positive result = 40/800 = 1/20

<table>
<thead>
<tr>
<th></th>
<th>Have the disease</th>
<th>Don’t have the disease</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Positive</td>
<td>160</td>
<td>40</td>
<td>200</td>
</tr>
<tr>
<td>Negative</td>
<td>0</td>
<td>760</td>
<td>760</td>
</tr>
<tr>
<td>Total</td>
<td>160</td>
<td>800</td>
<td></td>
</tr>
</tbody>
</table>

Probability of a positive result that is wrong
\[
\frac{40}{200} = \frac{1}{5}
\]

\[
D+P = 0.2 \times 1 = 0.2
\]
\[
D+N = 0.2 \times 0 = 0
\]
\[
ND+P = 0.8 \times 0.05 = 0.04
\]
\[
ND+N = 0.8 \times 0.95 = 0.76
\]
Probability of a false positive = 0.04

\[
\frac{200}{840} = \frac{1}{21}
\]
Medical Testing

“Do not miss the opportunity to discuss the surprise element of this task. If a patient goes to their doctor and gets a positive test, the chances of it being wrong are much lower in Country A (0.17) than in Country B (0.71)!

This is known as the ‘false positive’ paradox. The probability of a false positive depends not only on the accuracy of the test, but also on the characteristics of the sample population.”
Tools to support Professional Development
New challenges for the teacher

**Transmission**
- **Maths:** body of knowledge to cover
- **Learning:** individual listening and imitating
- **Teaching:** linear explaining

**Discovery**
- **Maths:** students create maths alone
- **Learning:** individual exploration and reflection
- **Teaching:** provide stimulating environment to explore, sequences activities and facilitates.

**Connectionist**
- **Maths:** teacher and students **co-create maths**
- **Learning:** collaborative learning through discussion
- **Teaching:** challenges, non-linear dialogue exploring meanings and connections.
Why tools for professional development?

• Shortage of PD leaders with the skills

• Designing PD that:
  • actually changes classroom practice
  • is cost-effective in teacher time

is a challenging design problem.

Key elements

• Activity based – active learning by teachers
• On-going – lifetime development
• In a framework – TRU

needs to be based on well-engineered materials
“The Sandwich Model”  Bowland Math, MAP

Modules to cover the major pedagogical challenges:

The model is a three part “sandwich”:

• **Introductory session:**
  Teachers work on problems, discuss pedagogical challenges they present, watch video of other teachers using these problems and plan lessons.

• **Into the classroom:**
  Teacher teach the planned lessons.

• **Follow-up session:**
  Teachers describe and reflect on what happened, discuss video extracts, and plan strategies for future lessons.
Strategic and structural design: some issues
What are they?

Strategic design

• focuses on the design implications of the interactions of products with the system they aim to serve.
• Important because of the many wonderful lessons, assessment tasks, and professional development activities that are never seen, while mediocrity is widespread

Structural design

• focuses on product structures that promise power in forwarding the strategy

See Educational Designer, Issue 3 – ISDDE online journal
Strategic design

Look for ‘leverage’ points “Why should they change?”

- WYTIWYG: examinations can be powerful levers
  Work to improve the exams

- Alignment: avoid mixed signals.
  Harmonise policy documents, exams, curriculum materials, and professional development

- Influence policy, whenever you can. See later
  Not easy. Focus on 'their' problems; offer win-win solutions

Plan pace of change

- How big a change can teachers carry through next year – given the support available? Big Bangs fizzle.
  Gradual change works – cf medicine
How can we help school systems change?

Now, to the new challenge:

Can we develop effective system-level tools?

e.g. Mathematics Improvement Network project – tools include:

System coherence health check

• Tool for system leaders to check proportions of novice, apprentice and expert tasks in their curriculum, assessments, PD

Principal’s classroom observation tool

• To help non-math-ed people pick out important things in the classroom (c.f. “quiet class at work”)
Structural design

Replacement units realise gradual change
Materials to support a few weeks new teaching

“The Box Model” realises alignment, integrating
• Task exemplars + teaching materials + DIY PD materials

Software microworlds support investigation
• Teachers and students naturally shift roles

“The Sandwich Model” supports activity-based PD
• Teachers face issues – teach lesson – reflect on what happened

The exponential ramp supports access to rich tasks
• “Apprentice tasks” that bridge from exercises to “expert tasks”
more design strategies

Identify target groups – notably teachers
  • Who do we need this to work for? Not just the enthusiasts!
    “Second worst teacher in your department” works well

Distribute design load
  • How much guidance shall we give to teachers?
    Offer detailed guidance when you are better placed to do so

Exemplars communicate
  • Descriptions will always be interpreted within experience.
    Task sets communicate vividly, video too.

PD should be task/activity based
  • Discussion-to-classroom gap.
    Teachers, too, learn experientially
Designing Tests
A Case Study
Facts: High-stakes assessment

Assesses student performance across task-types included

Exemplifies performance objectives
  • test tasks are assumed to exemplify the learning goals, so they effectively replace the policy documents

Determines the pattern of teaching and learning
  • Teachers ‘teach to the test’

So taking learning goals seriously implies designing tests that meet them:
  • “Tests worth teaching to”

that enable all students to show what they can do across the full range of learning goals
Strategic design: testing disasters

The “tests are just measurement” fallacy
  • In fact they dominate teaching and learning

Accepting cheap limited “proxy tests”:
  • eg multiple choice, computer adaptive
  They narrow learning, waste time on irrelevant test-prep

Content criterion-based testing drives down standards
  • It forces you to test the bits separately

Shell Centre has provided ways to avoid/mitigate these effects (TSS, NTPS, BAM, MAP)
Pushback

Fear of time, cost, litigation, .... anything new

Psychometric tradition and habit:
- testing is “just measurement”
- focus on statistics, ignoring systematic error ie not measuring what you’re interested in

Overestimating in-house expertise
- principles fine; tasks don’t match them

Good outcomes depend on close collaboration
- of assessment folk, math folk, outside expertise
England age 16: new Assessment Objectives

are very encouraging

35-45%: recall math knowledge, carry out routine procedures...

30-40%: reason and communicate, develop math argument, with substantial chains of reasoning...

20-30%: apply math knowledge and reasoning, modelling, solve non-routine problems, make connections across domains,

However, the actual tests .......
ISDDE

Goals – and progress so far

• Build a design community – it now exists
• Raise standards – real progress, learning together
• Increase influence on policy – central challenge now

If you design and develop tools for others to use you should consider applying for a Fellowship: see isdde.org
and read Educational Designer

Next world conference is

• 2019 Pittsburg, USA September 16-19
Why do classrooms change so slowly?
Claim

Many of us know how to enable typical teachers to teach much better mathematics much more effectively.

None of us know how to lead school systems to make the changes needed for this to happen on a large scale.

That is the central challenge of our time: Policy makers are part of the system.
The policy makers world

“If I want to talk to Education, who should I call?”

Politicians have many pressures:

- **Time** – busy lives with colleagues, civil servants, media
- **Pressure** – cabinet, party, lobbyists, media
- **Procedures** – must follow protocols
- **Money** – never enough, priorities compete
- **Timescales** – days > months education too slow

But also

- Technical naïvete – or arrogance
- Denial of expertise

Why?
The world of educational practice

The communities:

**Teachers** – their practice is key to improvement
- current demands are already more than enough
- pressure to focus on ‘measurables’ – test scores
- They have lives to live

**School leadership** – middle management ‘sandwich’
- again measurables – test scores, government inspections
- Resources v needs (class size!)

**Government** – managing the system
- Regional variations
- Resources
- Media pressure…..
The education research world

“If you so smart, how come you ain’t rich?”

Insight-focused in the ‘science’ tradition, but it values:

- new ideas over reliable research
- new results over replication and extension
- trustworthiness over generalizability
- small studies over major programs
- personal research over team research
- first author over team member
- disputation over consensus building
- papers over products and processes

Useful insight research needs big teams, long timescales
Engineering research in education

Methodology:

- **Research insights** from past research, other materials
- **Design:** imaginative design, combining creativity and experience
- **Systematic development** through
  - an iterative series of trials in classrooms, with
  - increasingly typical target groups of teachers and students,
  - revision via systematic use of rich and detailed feedback.

The way it’s done in research-based fields

engineering, medicine, agriculture
.... more specifically

“Fail fast, fail often” = rapid prototyping

Make feedback cost-effective
  • small samples ~5, enough to see general features
  • two or three iterations
  • rich data needs observation

Much more expensive than “authoring”
$3,000 per task, $30,000 per lesson – but can ensure that:
  • the activities work
  • the materials communicate, enabling users to succeed

But this cost is negligible within system running costs
Rebalancing research in Education

I argue** for better evidence on generalizability, needing:

“Big Education”

Other fields accept that

**Big problems in complex systems need big coherent collaborations**

using agreed common methods and tools, specifically developed for key problems of practice (CERN, Human Genome Project,...)

A challenge for the field

**“Mathematics Education Research: a strategic view”**

*Handbook of International Research in Mathematics Education*, 3rd Edition, Edited by Lyn English and David Kirshner
“Towards research-based education?

Executive Summary > Outline
1. Is there a problem?
2. The policy makers’ world
3. The educational research world
4. The world of educational practice
5. How could ‘the system’ work better?
6. The initial concerns addressed?

Appendix: What does good engineering look like?

How could ‘the system’ work better?
an explicit model, learning from medicine

Short term: treat each policy initiative as a design and development problem - needs expertise

Medium term: three strands & structures
• More systematic design and development NIED
• More complementary research NIER
• Sift for excellence: NIEE to support/guide:
  ▪ policy design
  ▪ practice in classrooms
Why should this work?

• R&D moves slowly – gets ahead of policy needs

• Builds on established practice, as in medical research <> practice

• Provides policy makers with a choice of well-engineered, well-proven initiatives

• Is cost-effective

Surprise! Interest from British government/science
Thank you
Exemplar Tools:
map.mathshell.org

‘Towards research-based education’


Contact
Hugh.Burkhardt@nottingham.ac.uk