For $\mathbb{E}^{n-2}$, $B^e_{\theta, \ast} \sim B^e_{\theta, \ast}$

**Cor:** $B^e_{\theta, \ast} \sim \Omega^{\infty, -1} MT\Theta$

**Objects:** $(n-2)$-connected $(M^{2n-1}, l_n)$

$$\varphi_k(M) = \begin{cases} 2 & k = 0, 2n \ll 1 \\
2 & k = n \\
0 & \text{else} \end{cases}$$

**Last step:** $\mathcal{D} \subset B^e_\theta$ is a full subcategory on obj. diffeo to $S^{2n-1}$

**Remark:** Not $B^e_{\theta, \ast}$ = full subc. on $(n-1)$-conn. $(M^{2n-1}, l_K)$

**Final Thm:** will be about $\Omega^{\infty, -1} MT\Theta = \Omega \Omega^{\infty, -1} MT\Theta$

so $\pi_0$ part of Cor. is "irrelevant".

**Today:** $B\mathcal{D} \to B^e_{\theta, \ast}$ is a weak equiv. onto a path comp of

**Next time:** Will use $\Omega B\mathcal{D} \sim \Omega^{\infty, -1} MT\Theta$ to prove main thm.

**Outline of $B^e_{\theta, \ast}$:** $\sim B^e_{\theta, \ast}$, $e \equiv n - 2$

$$|D^e_1| \longrightarrow |D^e_{1, e-2}|$$

- Easy, omitted
- Do surgery: "standard family"
- Cont. space of surgery
- Flag op. technique

**Two difficult steps**

- $(\ast)$ is a w.e.
- Construction of standard family:
  - at time $t$ makes obj. $e$-conn.
  - doesn't make connectivity worse at any time $t$. 
Summary of proof of last step:

\[ |D_s^1| \xrightarrow{\text{easy omitted}} |X^3| \]

\[ \text{as before} \]

\[ |D_{s,10}| \xrightarrow{\text{as before}} |D_{s,1}| \rightarrow |D_{n-2}^s| \rightarrow |X_{n-2}^s| \]

**Def.** \( D_p^s \subset X_{n-2}^s \in (a, \epsilon, W, l_w) \)

where \( W|a_1 \approx S^{2n-1} \) \( (3 \text{ diffeo}) \) \( \forall i \)

As before \( |D_{s,1}| = BD \)

\[ \ni \quad |D_{n-2}^s| = BE_{n-2,n-2} \]

is the subspace where:

\[ \text{if } a \in \bigcup_{i=0}^{p} (a_i - \alpha_i, a_i + \alpha_i) \]

is a reg. value of \( x_a : W = (a_0 - \epsilon_0, a_p + \epsilon_p) \rightarrow (0,1)^\infty \)

\( 3 \text{ diffeo } W|a \approx S^{2n-1} \)

**Def.** \( D_1^s \) consists of

\[ x = (a, \epsilon, W, l_w) \in D_{n-2}^s \]

together with \( e \in Y_q(x) = (q+1) \)-tuple of surgery data, each of which would make each \( W|a_1 \) into a manifold, diffeo to \( S^{2n-1} \)

Essentially as before, but replace "\( e \)-connected" with "diffeo to \( S^{2n-1} \)"

Will prove: \( |D_{s,1}| \rightarrow |D_{n-2}^s| \) is a weak equiv. onto a path comp. Then the diagram will imply

\[ |D_{s,1}| \approx \text{path comp. of } |D_{n-2}^s| \]
Remark: \( D^s = N_0 \mathcal{O} = \text{obj}(\mathcal{O}) = \text{BDiff}(D^{n-2}, s) \)

Two main differences from \( BE^{n-2, l} \rightarrow BE^{n-2, l-1} \)

(I) Proof of \( \text{ID}^s \rightarrow \text{ID}^{s-1} \) w. e. on comp

(II) Std. family: diag & htpy

(1) Last, actually proved: \( \text{ID}^s \rightarrow \text{ID}^{s-1} \) \( \forall p \)

using "flag split" technique.

Recall: \( \text{ID}^{s-1} \cong \text{ID}^{\infty} (\Omega^{\infty-4} \mathcal{M} \Theta) \cong \{ (M^{n-2}, l_M) \} \)

Path comp. of \( \text{ID}^{s-1} \) corres. to \( S^{2n-4} \) is the realization of the subspace of \( D^{n-2} \):

Those \( (a, \epsilon, w, l) : W_{\lambda a} \) is \( \Theta \)-bordant to \( S^{2n-4} \) \( \forall i \)

Will prove: \( \forall p \)

\( \text{ID}^s \rightarrow \text{ID}^{s-1} \) \( \text{b.w.e.} \) op. ex. d.

onto subspace, where

\( (W_{\lambda a}, l) \) is \( \Theta \) bordant to \( S^{2n-4} \).

Let \( x = (a, \epsilon, w, l) \in D^{n-2} \) be such a elt.

Need surgery data

\( \gamma_0(x) \) is surgery data to make each \( W_{\lambda a} \) into mfd
diff. to \( S^{2n-4} \)

\( \gamma_0(x) = \text{disj. } (p+1) \text{ tuples. } (\text{or disj. on cores}) \)

Why is \( \gamma_0(x) \) non-empty? We assume

\( M = W_{\lambda a} \)

Claim: There exists a bordism \( Q \), which is \((n-1)\)-conn.

Proof: Do surgery on the interior of \( O \): If only \((n-1)\)-conn

pick \( S^d \rightarrow Q \) gen. \( T\epsilon(Q) \), homotop to emb.

\( (\leq n) \)
Pick trivialization of normal bd of $S^l \times D^{n-l} \to Q$

Do surgery $\Rightarrow Q$ $\ell$-conn.

$2n \geq 6 \quad M \to Q$

$(n-2)$-conn $(n-1)$-conn.

so $(Q, M)$ is $(n-1)$-conn.

$\Rightarrow \text{H}^e (Q, M) = 0$ for $e \leq n-1$

$(Q, S^{2n-2})$ $(n-2)$-conn

$\Rightarrow \text{H}^e (Q, S^{2n-2}) = 0$ for $e \leq n-1$

$\text{emb. thm}$

$\Rightarrow \text{H}^e (Q, M) = 0$ for $e \neq n$. (And H is free)

So by the proof of the h-sph. thm:

(since $2n \geq 6$) Can find a Morse fn on $Q$ with only cnt. pts of index $n$

Pick metric on $Q$. Then the descending mfs of the cnt.

gives finitely many $S^{n-1} \times D^n \to M$

s.t. surgery gives mfs diff to $S^{2n-1}$

Main step in $Y_0(X)$ being non-empty.

Also need: Given finitely many $y_i \in Y_0(X)$, can find $y \in Y_0(X)$ disj. from $y_i$'s (at least disj. at cores).

To make disj., use "Whitney trick"
\[ \begin{align*}
\text{Diagram with symbols: } & a_1, a_{\text{inc}}, a_p \\
\end{align*} \]