Cornelia Drutu

*C*-algebras and Geometric Group Theory

In this mini-course I will overview various topics at the interface between *C*-algebras and Geometric Group Theory, from the Rapid Decay property to Kazhdan projections and various versions of amenability.

Ilijas Farah

Massive *C*-algebras and their Applications

Given a separable *C*-algebra $A$ one can define the asymptotic sequence algebra $\ell_\infty(A)/c_0(A)$. By taking further quotients, one obtains various other extensions of $A$, the most important of which are the ultrapowers. When $A$ is separable, the relation between $A$ and these ‘massive’ *C*-algebras is used to provide information about $A$ itself. Quite surprisingly, this approach is frequently more efficient than the direct study of $A$ itself. In these lectures, I will provide a friendly introduction to asymptotic *C*-algebras and ultrapowers, and show that in a certain precise sense the latter construction is, for all practical purposes, more general (i.e., more useful).

Ian F. Putnam

*C*-algebras and Dynamics

The main goal of the lectures is to provide an introduction to the connections between topological dynamical systems and *C*-algebras. This interaction has been fruitful for both fields, going back to the seminal work of Murray and von Neumann. On the one hand, *C*-algebras constructed from dynamical systems provide important examples. On the other, *C*-algebraic techniques and tools, particularly $K$-theory, provide interesting new insights into the dynamical systems.

The lectures will begin with the construction of *C*-algebras from groupoids, since this is at the heart of the interaction. We will briefly discuss $K$-theory, particularly for groupoid *C*-algebras and crossed products. We will also give a substantial list of topological dynamical systems, simply to provide a first insight into the types of behaviors that are possible, discussing the *C*-algebras briefly in each case. These include minimal systems, AF-equivalence relations, Smale spaces and examples from the field of aperiodic tilings.

Finally, we discuss applications of relative $K$-theory to the dynamics. The first example of such results was in the so-called orbit-splitting subalgebras of crossed products, which have played a key part in Elliott’s classification program, at least as it applies to crossed products. We will give some more recent results along these lines, with applications to such systems as the dynamics of flat surfaces and some problems in the Elliott classification scheme.