I will present a connection between quantum information and quantum permutation groups. Specifically, I will present a notion of quantum isomorphisms of graphs based on quantum automorphisms from the theory of quantum groups, and then show that this is equivalent to the previously defined notion of quantum isomorphism corresponding to perfect quantum strategies to the so-called isomorphism game. Moreover, we will see that two connected graphs $X$ and $Y$ are quantum isomorphic if and only if there exists $x \in V(X)$ and $y \in V(Y)$ that are in the same orbit of the quantum automorphism group of the disjoint union of $X$ and $Y$. This connection links quantum groups to the more concrete notion of nonlocal games and physically observable quantum behaviours.

We will see that this link allows us to use ideas and results from quantum information in order to prove new results about quantum automorphism groups. In particular, we will see that asymptotically almost surely all graphs have trivial quantum automorphism group. Furthermore, we will construct an infinite family of graphs which are quantum vertex transitive but fail to be vertex transitive, answering a question from the quantum group literature. (This talk is based on a joint work with Martino Lupini and David E. Roberson.)