RATIONAL EQUIVARIANT COHOMOLOGY THEORIES

Conjecture: For any compact Lie group there is a (small, explicit, calculable) abelian category $\mathcal{A}(G)$ of injective dimension rank(G) and a Quillen equivalence

$$G\text{-spectra}/\mathbb{Q} \simeq d\mathcal{A}(G).$$

More properly, there are various versions of the conjecture depending on how much structure the equivalence is supposed to preserve. In increasing order

(1) Adams Spectral sequence
(2) equivalence of homotopy categories
(3) Quillen equivalence (as stated)
(4) monoidal equivalence
(5) equivalence of E^1_∞ (i.e. naive-commutative) ring objects and modules over them
(6) equivalence of E^G_∞ (i.e. fully equivariantly-commutative) ring objects and modules over them

G	ASS	Ho	Quillen	Monoidal	E^1_∞	E^G_∞
Finite $SO(2)$ Tori	GM [12] G [8]	GM[12] G [8]Not Δ[22]	SchS[23] GS[18] GS[18]	K[19] BGKS[6]	BGK[5] BGK[4]	W[24]
$O(2)$ $SO(3)$	G[7] G[9]		B[1] K[20]			
Free Toral	 G[11]		GS [13, 14] BGK[3]			

B=Barnes
G=Greenlees
K=Kedziorek
M=May
Sch=Schwede
Sh=Shipley
W=Wimmer

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