

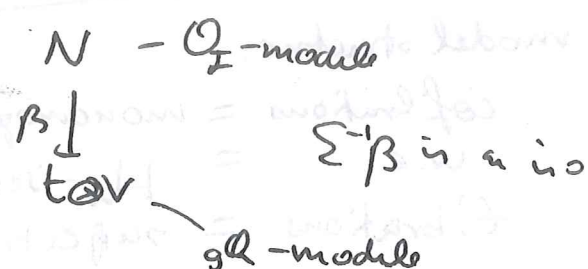
Barnes: Rational S^2 -Equivariant Ring Spectra

$$\mathcal{O}_F = \prod_{n \geq 1} \mathbb{Q}[c_n]$$

$$\Sigma^{-1} \mathcal{O}_F = \text{colim}_k \left(\prod_{i=1}^k \mathbb{Q}[c_i, c_i^{-1}] \times \prod_{j \geq k+1} \mathbb{Q}[c_j] \right) = t$$

$$\begin{matrix} \otimes \mathbb{Q} \\ S^2 = SO(\mathbb{Z}) \\ \prod = \Pi \end{matrix}$$

$\mathcal{A}(S^2)$ object



morphisms:

$$d\mathcal{A}(S^2) = \text{objects } X \xrightarrow{\partial} \Sigma X, \quad \partial^2 = 0$$

morphisms as in $\mathcal{A}(S^1)$ + commute with differentials

Monoidal structure:

$$\begin{pmatrix} M \\ \downarrow \\ t \otimes v \end{pmatrix} \otimes \begin{pmatrix} N \\ \downarrow \\ t \otimes w \end{pmatrix} = \begin{matrix} M \otimes N \\ \downarrow \beta \otimes \gamma \\ t \otimes (v \otimes w) \end{matrix}$$

This has an internal function object

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Let $\hat{\mathcal{A}}$ be $\mathcal{A}(S^2)$ without the condition on β .

$$\mathcal{A}(S^2) \xrightleftharpoons[\Gamma]{\text{inc}} \hat{\mathcal{A}} \quad \Gamma \circ \text{inc} = \text{id}$$

$$F \left(\begin{pmatrix} M \\ \downarrow \\ t \otimes v \end{pmatrix}, \begin{pmatrix} N \\ \downarrow \\ t \otimes w \end{pmatrix} \right) = F \left(\begin{pmatrix} P \\ \downarrow \\ t \otimes \text{Hom}_{\mathbb{Q}}(v, w) \end{pmatrix}, \begin{pmatrix} \text{Hom}_{\mathcal{O}_F}(M, N) \\ \downarrow \\ \text{Hom}_{\mathcal{O}_F}(M, t \otimes w) \end{pmatrix} \right)$$

$$0 \rightarrow \pi_*^{\text{cl}}(X) \oplus \pi_*^{\text{cl}}(Y) \rightarrow \pi_*^{\text{cl}}(X \wedge Y) \rightarrow \Sigma \text{Tor}_{\mathcal{A}}(\pi_*^{\text{cl}}(X), \pi_*^{\text{cl}}(Y)) \rightarrow 0$$

\cong

$$\pi_*^{\text{cl}}(X) \xrightarrow{\mathcal{S}^2 \mathcal{F}_Q} \pi_*^{\mathcal{S}^2}(X \wedge_{\text{DEF}_+})$$

$$\pi_*^{\mathcal{S}^2}(X \wedge_{\text{DEF}_+} \mathbb{1}_{\text{EF}}) = \text{tor} \pi(\mathbb{1}^{\mathcal{S}^2} X)$$

Model structures on $\mathcal{A}(\mathcal{S}^2)$:

Injective model structure:

- cofibrations = monomorphisms
- w.e. = H_x -isos
- fibrations = surjections with injective kernel

This is not monoidal

Every object is cofibrant

$\otimes X$ should preserve cofibrations

Exercise!

Definitions: We say $X = \begin{pmatrix} M \\ \downarrow \\ \text{tor} V \end{pmatrix}$ in $\mathcal{A}(\mathcal{S}^2)$ is strongly dualizable if any of the following equivalent conditions hold:

- $\text{Hom}_{\mathcal{A}} \left(\begin{pmatrix} M \\ \downarrow \\ \text{tor} V \end{pmatrix}, \begin{pmatrix} N \\ \downarrow \\ \text{tor} W \end{pmatrix} \right) \cong \underbrace{\text{Hom}_{\mathcal{A}} \left(\begin{pmatrix} M \\ \downarrow \\ \text{tor} V \end{pmatrix}, \begin{pmatrix} \mathcal{O}_F \\ \downarrow \\ \text{tor} A \end{pmatrix} \right)}_{DX} \otimes \begin{pmatrix} N \\ \downarrow \\ \text{tor} W \end{pmatrix}$ for all $\begin{pmatrix} N \\ \downarrow \\ \text{tor} W \end{pmatrix}$ in $\mathcal{A}(\mathcal{S}^2)$

- $DDX \cong X$
- N is finitely presented in \mathcal{O}_F -modules and B is injective
- $\cong \text{Hom}_{\mathcal{O}_F}(M, N)$
 $\text{tor} \text{Hom}(V, W)$

Theorem (B): There is a symmetric monoidal model structure on $\text{dMod}(S^1)$ with w.e.s the H_* -isos and gen. cofibrations:

$$I = \left\{ S^{n-1} \otimes_{\mathbb{Q}} P \rightarrow D^n \otimes_{\mathbb{Q}} P \mid P \text{ is in a stel. of the strongly dualizable} \right\}$$

$$S^{n-1} \begin{matrix} \vdots \\ 0 \\ \vdots \\ \mathbb{Q} \\ \vdots \\ 0 \\ \vdots \end{matrix} \rightarrow \begin{matrix} \vdots \\ \mathbb{Q}^n \\ \vdots \\ \mathbb{Q}^{n-1} \\ \vdots \\ 0 \\ \vdots \end{matrix} \otimes_{\mathbb{Q}} D^n$$

Proof: Smith's theorem.

We are in a Grothendieck category and the H_* -isos are accessible.

— we need I -inj $\subseteq H_*$ -isos
 This follows as there are enough dualizables.

$$\text{dMod}(S^1)_{\text{dual}} \xrightleftharpoons{\text{id}} \text{dMod}(S^1)_{\text{inj}}$$

Joint with Greenlees, Kedziorek, Shipley

$$\text{SO}(2)S^1P_{\mathbb{Q}} \xrightleftharpoons[\text{cell-}]{\text{cell-}} \left(\begin{array}{c} L_{\mathbb{E}\mathbb{F}} S^1S^1P_{\mathbb{Q}} \\ \downarrow \wedge \text{DEF}_+ \\ \text{DEF}_+ \text{-mod} \xrightarrow{\text{id}} L_{\text{DEF}_+ \wedge \mathbb{E}\mathbb{F}} \text{DEF}_+ \text{-mod} \\ \uparrow S^1S^1P_{\mathbb{Q}} \end{array} \right)$$

cell principle

Isotropy separation:

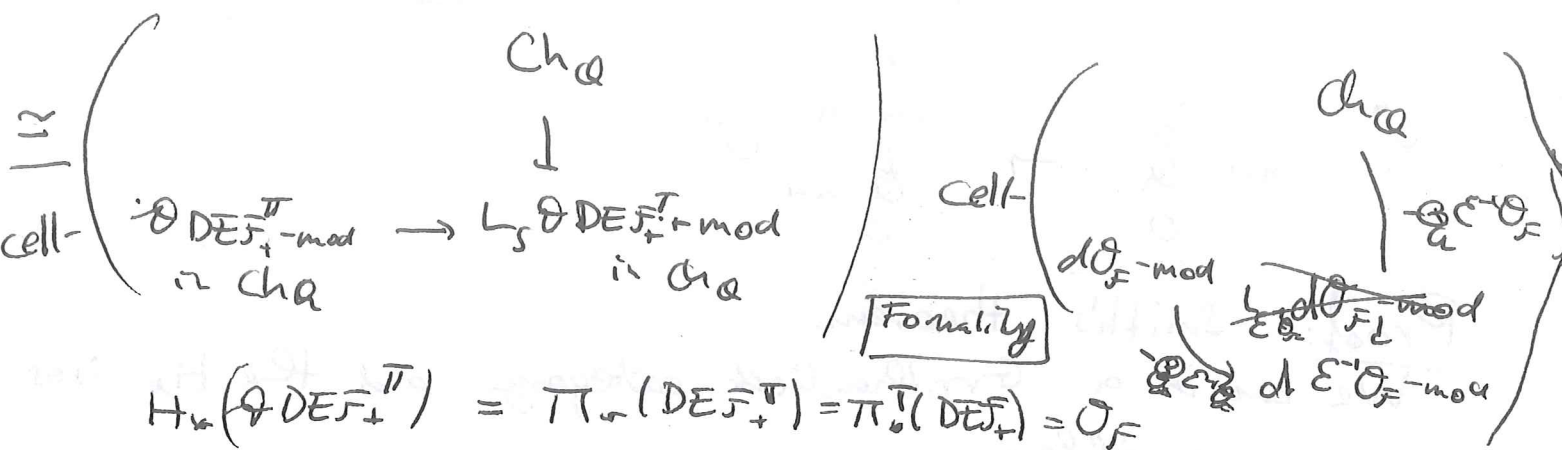
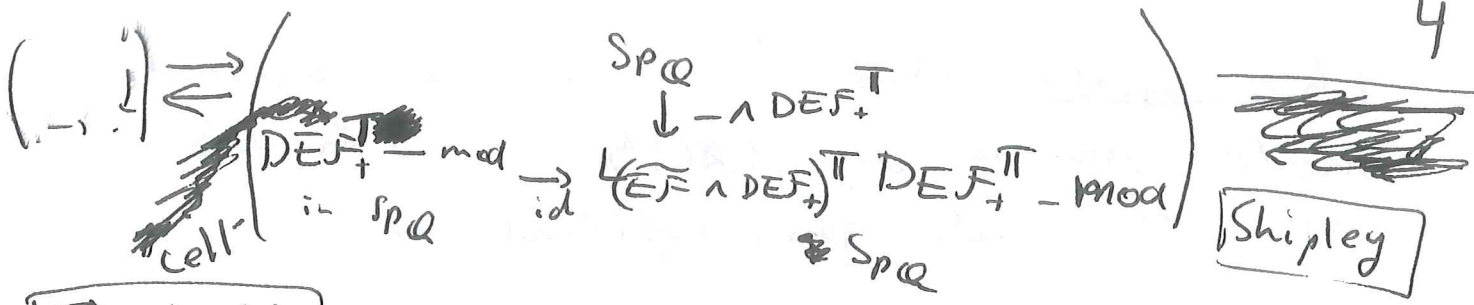
$$(X, M, K)$$

$$X \rightarrow M \leftarrow K \wedge \text{DEF}_+$$

$$\begin{array}{ccc} \mathcal{S} & \longrightarrow & \tilde{\mathbb{E}\mathbb{F}} \\ \downarrow & \lrcorner & \downarrow \\ \text{DEF}_+ & \longrightarrow & \text{DEF}_+ \wedge \tilde{\mathbb{E}\mathbb{F}} \end{array}$$

$$\tau \longmapsto (\text{DEF}_+ \wedge \tau, \text{DEF}_+ \wedge \tau, \tau)$$

$$\text{in } S^1S^1P$$



$H_n(\otimes DEF_+^{\pi}) = \pi_n(DEF_+^{\pi}) = \pi_n^{\pi}(DEF_+) = \mathcal{O}_F$

$\mathcal{A}(\mathcal{B}^{\pi})_{dual}$

