A student’s perception of CAS-related sociomathematical norms surrounding teacher change in the classroom
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In this paper, we investigate the use of CAS in Danish upper secondary school from a norms perspective. The construct of sociomathematical norms is used to understand the potential influence that CAS may have on the rules (and values) in the mathematics classroom. We focus on a situation of teacher change, where the different teachers enact different norm sets related to students’ CAS use in their mathematical work. We zoom in on one student’s perception of this, and find that the teachers’ CAS policies have a direct influence on how students’ navigate in their mathematical work. In particular, the alignment between teachers’ endorsed and enacted norms as well as the extent to which the norms impose judgement on the students’ behalf in relation to CAS use play a major role in this respect.

Keywords: CAS, CAS policy, socio-mathematical norms, teacher change.

INTRODUCTION

Computer Algebra Systems (CAS) play an important role in Danish upper secondary school, as is the case in several other countries. It has been argued for more than 15 years that the challenges posed by CAS to the organization of mathematics teaching is not one that has been called for by the school topic itself, but rather a consequence of the development of technologies for non-didactical purposes (Trouche, 2005). The didactical difficulties and possibilities of CAS use have been widely documented (e.g. Hoyles & Lagrange, 2010; Jankvist & Misfeldt, 2015; Weigand, 2014), but teachers are largely left to develop their own ideas and understanding of how to use CAS in teaching (e.g. Jankvist, Misfeldt & Marcussen, 2016). Hence, different teachers—at least in Denmark—work with these tools in different ways, enacting different values and norms about for example the subject of mathematics, its teaching, and the use of technology, and not least the relation between these.

In this paper, we ask how students experience and make sense of various teachers’ different views on the role of CAS. In order to see this phenomenon more clearly, we investigate how one student—Emil—experienced and tried to navigate between various teachers’ different CAS-related norms. More precisely, from a teacher focusing on the students’ ability to work with paper and pencil to another teacher focusing on correct and efficient CAS use, and again to a third teacher who believed that the students should decide themselves when and when not to use CAS. We apply the term CAS policy to articulate teachers’ expectations about students’ CAS use in their mathematical work. We use the construct of sociomathematical norms to understand the influence of such CAS policies on students’ learning and their possibilities to participate in the mathematical activities.
SOCIO-MATHEMATICAL NORMS

Sociomathematical norms were observed and named by Yackel and Cobb (1996), who in a teaching situation noticed that aspects which could neither be described as purely mathematical norms nor purely as classroom social norms were in play. Yackel and Cobb defined sociomathematical norms as “normative aspects of mathematical discussions specific to students’ mathematical activity” and describe the difference to social norms as:

The understanding that students are expected to explain their solutions and their ways of thinking is a social norm, whereas the understanding of what counts as an acceptable mathematical explanation is a sociomathematical norm. Likewise, the understanding that when discussing a problem students should offer solutions different from those already contributed is a social norm, whereas the understanding of what constitutes mathematical difference is a sociomathematical norm. (Yackel & Cobb, 1996, p. 461)

Sociomathematical norms are negotiated between the students and the teacher, and may thus vary from classroom to classroom. This negotiation builds on already “taken-as-shared” perceptions within the classroom, and as such they are:

… intrinsic aspects of the classroom’s mathematical microculture. Nevertheless, although they are specific to mathematics, they cut across areas of mathematical content by dealing with mathematical qualities of solutions, such as their similarities and differences, sophistication, and efficiency. Additionally, they encompass ways of judging what counts as an acceptable mathematical explanation. (Yackel & Cobb, 1996, p. 474)

In the study described by Yackel and Cobb, a sociomathematical norm is negotiated where an acceptable mathematical explanation must describe actions performed on mathematical objects. Hence, explanations and justifications are themselves made the objects of reflection. According to Levenson, Tirosh and Tsamir (2009), there are three kinds—or aspects of—sociomathematical norms that should be taken into account: teachers’ endorsed norms; teachers’ and students’ enacted norms; and students’ perceived norms. Based on classroom studies, Levenson et al. noticed that even when the observed enacted norms were in agreement with the teachers’ endorsed norms, students may not have the same perception of these norms.

EDUCATIONAL SETTING OF THE CASE STUDY

Denmark has three different types of upper secondary school programs: the classical stream, the technical stream, and the business stream. Danish upper secondary school is usually three years, and students may take mathematics at one of three levels (C, B or A), depending on the number of years they follow it, e.g. A-level is mathematics for all three years. CAS were introduced into the upper secondary streams in 2005 and is now a mandatory part of the national written assessments. For the technical stream, the ministerial orders for mathematics A-level state:

The student works with CAS tools and other mathematical software, so that the student becomes familiar with syntax, terminology and application of at least one mathematical software. Over the course of the program, the digital tools may be increasingly applied for:
modelling; visualizations; geometric investigations; repetitive calculations; complex symbolic manipulations and calculations; numerical calculations; documentation and communication of results. (UVM, 2013)

In some Danish upper secondary schools, CAS are not introduced until after Christmas in the first year, i.e. the first semester follows a more traditional paper-and-pencil approach, while other schools provide students with a license to a given CAS-tool from day one. Even if schools do offer a CAS license from day one, individual teachers may still choose to wait until later in the first year. Furthermore, how exactly CAS is then introduced and used is often left entirely up to the individual teachers, subject of course to equally independent decisions and discourses of a given textbook system.

Our case stems from an A-level mathematics class at the technical stream; one which experienced four different mathematics teachers within their three years. We focus on the student Emil (see also Iversen, 2014; Iversen, Misfeldt & Jankvist, accepted) and his perception of the various teachers’ CAS policies. Our case description and analysis solely builds on an interview with Emil and excerpts from his hand-in assignments, corrected and commented by the different teachers.

CASE: THE UPPER SECONDARY SCHOOL STUDENT EMIL

The first of Emil’s four teachers, Teacher 0, was only in the class for a very short time at the beginning of the first year, for the reason of which we do not—and neither did our case student, Emil—take her influence on the class into account. Emil provided the following description of the Teachers 1, 2, and 3, and their approaches to the use of CAS (all quotations are translated from Danish):

Emil: The first teacher [1] made it very clear that the purpose of using sketches was that the teacher/reader should be able to see what was going on in our minds. He “punked” us about that; lots of sketches, and they had to be good, so he could follow our way of thinking. The next teacher [2] talked a lot about us using our tool in a correct manner; that now we were past the point, where we had to explain everything; that now we had to use it [i.e. CAS] and see that we can come from A to B faster; and that we had to solve tasks. The third teacher [3] has been a bit of a mix, saying we had to use the tools more limitedly, and that she also wanted us to be able to do some mathematics. (Emil, May 9, 2012)

Neither Teacher 0 nor Teacher 1 spend time introducing CAS to the students. Teacher 2 however did, more precisely to Maple. Emil explained:

Emil: Yes, almost immediately we got this new software, Maple, for the computer. Then we spent a few lessons learning the basics about it. You can say that Maple can do more compared to the handheld calculator, because when we deal with stuff like, for example, rotation around a fixed axis, the calculator can’t sketch this. The big change from the first teacher [1] to the second teacher [2] was going from doing everything by hand to having to solve everything on the computer now.
Iversen: Everything? Like the hand-in assignments, or also during lessons or what?

Emil: I would go so far as to say everything. [...] 

Emil: So, while with the first [Teacher 1] it was like, say, if we had to isolate in relation to something, then we had to do it by hand, then [with Teacher 2] it was just “Solve” [in CAS]. You still had to write down a little about what you were doing, but you didn’t need the long steps of calculations, you could just use the computer now. (Emil, Sept. 9, 2011)

Figure 1 illustrates a hand-in assignment by Emil at the time of the teacher change to Teacher 2, i.e. at the beginning of the second year (August, 2010).

Figure 1: Homework assignment from Emil’s second year of upper secondary school.

Notice the text at the bottom “eller i maple” meaning “or in Maple”.

The task reads: “A particle moves in the plane, so that it at time $t$ is located in the point with coordinates $f(t)$, where $f(t) = \ldots$ Find those points of time $t$ for which a)... b)... c)...” We are interested in Emil’s answer to question a), in which he used that the dot product of the functions $f'(t)$ and $f''(t)$ should be zero. Emil answered question a) of the task by first doing a rather long and—at least at this level of education—somewhat complex calculation (bottom, left hand side) and immediately after solving the same task with one line of Maple code. What is interesting is his need for including both solutions as his answer to the question. Email elaborated:
Iversen: Okay, so what did you think about the shift from having to do everything by hand and then having to use... [CAS]?

Emil: It was very intense. It felt much easier to get the computer to do it. You saved a lot of time. In a way you felt that you had been ridiculed; that you had to do so much with the first teacher [1], and then this intense shift... but the good thing always was—not to jump to the third teacher [3] [...]—yes, because with the first teacher [1], you always knew what to do. There was no question that you had to do everything by hand. With the second teacher [2] I knew that I had to do as much as possible by means of the computer [...] he didn’t want to see the intermediate results. (Emil, Sept. 9, 2011)

As evident from the above, once Emil had figured out the CAS policy of Teacher 2, it became equally clear to him what was expected as it was previously with Teacher 1. But as hinted to with the saying of “not to jump to the third teacher...”, something about Teacher 3’s CAS policy was less clear to him:

Iversen: If we take the thing with Maple [...] you’ve really taken this to heart and used it a lot. In the hand-in assignments I’ve looked at, there are a couple of times where she [Teacher 3] comments something like “phew”, when there are long expressions or long commands in Maple or something like that...

Emil: I also got some saying “I think I can almost follow what is going on” [...] Iversen: Yes, and what exactly do you think about that?

Email: I know that my teacher [3] isn’t very... well, I wouldn’t call her old fashioned. That would be wrong. Maybe she just isn’t very fond of using Maple. [...] Iversen: What do you think about the impression it makes on a reader, for example, that your mathematical texts include these long command sequences? [...] Emil: My mom thinks it looks advanced...

Iversen: So, not to over interpret, but could it be somewhat the same with your teacher [3], when she writes “phew” and “I think I can follow it” and so on?

Emil: It could be that it may seem a bit “overkill”—or with a Danish saying; “to shoot sparrows with cannons”—to use Maple. (Emil, May 9, 2012)

From lower secondary school, Emil was used to a traditional paper-and-pencil approach to the teaching and learning of mathematics. The first “shift” for Emil concerned having to hand in all mathematics assignments in electronic form in upper secondary school—a rule installed by Teacher 0. With Teacher 1, Emil again experienced a traditional approach, since the use of technology was limited to a text editor, e.g. MS Word and its equation functionalities, and software to draw sketches, e.g. Graph. With Teacher 1, the CAS functionalities of the handheld TI-89 were only to be used to check results obtained by paper-and-pencil methods.
With Teacher 2, however, Emil experienced a change in how to apply CAS in the hand-in assignments. Teacher 2 did not care about intermediate steps and detailed calculations. Rather he endorsed (or even required) that everything had to be done on the computer, and he considered correct use of Maple as a key competence for the students. Hence, the focus for evaluation of the students’ written products changed from a focus on providing detailed algebraic calculations, to a focus on how to address the problem with the tool and resources in an efficient and correct manner. According to Emil, Teacher 2 expressed that the students should not explain every little detail of the calculations but instead focus on correct application of Maple.

Teacher 3 had a more liberal, balanced and almost unengaged approach to the use of CAS. Students could choose to use CAS, when they wanted to and when they could argue for its meaningfulness. Emil described this as a “mix” of the two previous teachers’ policies, and he also suggested that Teacher 3 did not possess a deep knowledge of advanced CAS tools, and that she often considered it to be “overkill” to apply such tools. Even though this teacher had a liberal approach to CAS, we can see from her comments to some of Emil’s written assignments that she did consider it interesting whether certain algebraic steps had been conducted with or without the aid of CAS (Iversen et al., accepted).

EMIL’S EXPERIENCE OF ENDORSED AND ENACTED NORMS

Overall, the norms, rules and regulations around CAS changed from suppressing CAS arguments (Teacher 1) to endorsing such arguments (Teacher 2), to a more balanced approach suggesting openness about whether or not CAS should be applied in a given situation, and a demand for active argumentation on the students’ behalf as to why they used CAS for a given task (Teacher 3).

Emil experienced the sociomathematical norms endorsed by Teacher 1 as aligned with the enacted norms of Teacher 1. Hence, at the time of Teacher 1 there was some coherence between Emil’s perceived norms and his enacted norms in relation to CAS use. However, a discrepancy occurred between Emil’s experience of the endorsed norms of Teacher 1 and Teacher 2, since they appeared to have rather different views on the role of CAS in the teaching and learning of mathematics. Both Teacher 1 and Teacher 2, respectively, appeared to have their own endorsed and enacted norms aligned. For Emil, however, the new norm set of Teacher 2 challenged his perceived norms due to the teaching of Teacher 1. We see this from his hand-in assignment (cf. figure 1) at the time when Teacher 2 had recently taken over the class. Here Emil provided solutions that potentially could satisfy the endorsed and enacted norms of Teacher 1 as well as those of Teacher 2. While Teacher 1 emphasized cognitive strategies and algebraic skills, Teacher 2 was much more focused on efficient problem solving and correct use of Maple. Furthermore, the overall problem solving approach was highly valued by Teacher 2 who was less focused on detailed aspects of computation and argumentation.
In time, Emil aligned the problem solving in his assignments to the new norm set of Teacher 2. In fact, Emil expressed that although the shift from Teacher 1 to Teacher 2 was very “intense” in relation to the use and role of CAS, it was still clear to him what was expected of him—both from Teacher 1 and from Teacher 2. This, we believe, has to do with Emil’s experience of alignment between the endorsed and the enacted norms of Teacher 1 and Teacher 2, respectively. This appeared to make it easier for Emil to align his own perceived norms with his enacted norms.

Teacher 3 endorsed yet a new set of norms, which entailed that CAS should be used when it makes sense to use it in a given mathematical situation. As far as we can tell Teacher 3 also enacted norms according to this (Iversen, 2014). Still, Emil apparently found it difficult to decode the norm set of Teacher 3. But why is this, when an alignment of the endorsed and enacted norms of Teacher 1 and Teacher 2, respectively, seemed to make this easier for Emil? It appears that such an alignment may be thought of as a necessary although not sufficient condition. Hence, it makes sense to ask why the sociomathematical norms that Teacher 3 brought into the classroom were so difficult for Emil—a high-performing mathematics student—to perceive? One explanation may be that the norm sets of Teachers 1 and 2 in a sense were rather binary—never use CAS and always use CAS—while the norm set of Teacher 3 imposed upon the students to make an actual judgement of when it is needed and when it is not needed to use CAS. Performing such judgement is surely more demanding on the students and requires them to develop competences to do so. Eventually Emil did seem to make sense of Teacher 3’s sociomathematical norms as indicated by his statement that she did not appreciate when you “shoot sparrows with cannons”—which is Danish for “take not a musket to kill a butterfly”—meaning of course that there is no need to use a powerful CAS tool to do something which you could equally easy, or maybe even easier, do by hand.

**FINAL REMARKS ON EXPERIENCED CAS POLICIES**

We have witnessed how an upper secondary student had to learn to navigate between different teachers’ varying CAS-related norms. In particular, the alignment between teachers’ endorsed and enacted norms as well as the extent to which the norms imposed judgement on the students’ behalf played a major role in this respect.

Due to changing ministerial orders in relation to CAS use in Danish upper secondary school, and due to lack of alignment between these orders, textbook writes’ interpretations of the orders, local school policies and teachers’ own policies, students are bound to “feel like a fish out of water”. In order to better understand this problem of teachers’ CAS policies we applied the construct of sociomathematical norms. We find this construct to have been productive, not least due to the explanatory power of considering (students’ experiences of) teachers’ endorsed and enacted norms and the students’ own enacted and perceived norms. Furthermore, established theoretical constructs, such as that of sociomathematical norms, make up relevant lenses to understand how different teachers’ different CAS policies shape the experience of students’ participation in the classroom, not least in a situation of teacher change.
Finally, we propose that in order to account more deeply for situations regarding teachers’ CAS policies, the use of sociomathematical norms could be augmented with theoretical constructs considering teachers’ mathematics-related values and beliefs related to technology. Such theoretical *bricolage* considerations shall be part of our future endeavours.

**REFERENCES**


