Stratifications and duality in modular representation theory

Lecture plan

1. Support for modules over commutative rings (Srikanth)

2. Triangulated categories with ring actions (Henning)

3. The stable module category of a finite group (Henning)

4. Generic points in commutative algebra (Srikanth)

5. Passage to closed points in representation theory (Srikanth)

6. Auslander-Reiten formula and Tate duality (Henning)

7. Local duality for modular representations (Srikanth)

8. Pi-points for modules over finite groups (Henning)

9. Elementary abelian groups (Srikanth)

10. Stratification for modular representations (Henning)

Abstracts

1. Support for modules over commutative rings

This lecture will be concerned with the structure theory of injective modules over commutative noetherian rings, à la Matlis. It will cover also Gabriel's classification of the Serre subcategories of the module category.

2. Triangulated categories with ring actions

For a compactly generated triangulated category with a central action of a graded commutative ring, local cohomology functors are introduced. This yields a notion of cohomological support.

3. The stable module category of a finite group

For a finite group G and a field k, the stable module category of G, denoted StMod kG, is a compactly generated triangulated category with a canonical action of the cohomology ring. In particular, local cohomology functors are defined. The running example is the Klein 4-group in characteristic two, where all representations can be described.

4. Generic points in commutative algebra

Given a finitely generated algebra A over an algebraically closed field k, Hilbert's Nullstellensatz identifies a maximal ideal of A (in other words, a closed point of Spec A) with the functions vanishing at a point in affine space. Zariski and Weil introduced a construction of a generic point to treat non-closed points in the spectrum of an affine algebra in a similar fashion. This lecture will be devoted to explaining this construction, and an analogue for graded algebras that plays a crucial role in our approach to duality and stratification.

5. Passage to closed points in representation theory

This lecture will be about a descent principle for the stable module category of a finite group. As will be explained in later lectures StMod kG can be stratified by the action of its cohomology ring. The central result relates the category of p-local p-torsion modules in StMod kG, to the modules supported at a closed point in the variety of KG, where K is a suitably chosen transcendental extension of k.

6. Auslander-Reiten formula and Tate duality

The Auslander-Reiten formula for modules over finite dimensional algebras is proved. This yields Tate duality for modules over a group algebra.

7. Local duality for modular representations

This lecture will be dedicated to a proof of a result that can be interpreted to mean that StMod kG is Gorenstein, when viewed as a category over its cohomology ring. One corollary is that the subcategory of compact objects in the p-local p-torsion subcategory of StMod kG has Serre duality.

8. Pi-points for modules over finite groups

The notion of a π-point is introduced. Using an appropriate equivalence relation, the π-points are identified with the projective variety of the cohomology ring. The π-points give rise to alternative definitions of support and cosuppport. For this notion of support, a tensor product formula is proved. Analogously, a formula for the cosupport of a function object is established.

9. Elementary abelian groups

The aim of this lecture will be to outline a proof of the classification of localising subcategories of StMod kE, where E is elementary abelian. A bijection is established between the tensor ideal localising subcategories of StMod kG, for a finite group G, and the subsets of the projective variety of the cohomology ring.

10. Stratification for modular representations

The stratification statement from the previous lecture will be established for all finite groups. One can think of this as stratification of the module categories that ‚categorifies' the Quillen stratification of the group cohomology. A crucial ingredient of the proof is the fact that cohomological support and cosupport coincide with π-support and π-cosupport, respectively.

References

J. L. Alperin, L. Evens, Varieties and elementary abelian groups, J. Pure Appl. Algebra 26 (1982), 221–227.

G. S. Avrunin, L. L. Scott, Quillen stratification for modules, Invent. Math. 66 (1982), 277–286.

D. J. Benson, Representations and Cohomology I: Basic representation theory of finite groups and associative algebras, Cambridge Studies in Advanced Mathematics, vol. 30, Cambridge University Press, 1991.

D. J. Benson, Representations and Cohomology II: Cohomology of groups and Cambridge Studies in Advanced Mathematics, vol. 31, Cambridge University Press, 1991,

D. J. Benson, J. F. Carlson, and J. Rickard, Complexity and varieties for infinitely generated modules. II, Math. Proc. Cambridge Philos. Soc. 120 (1996), 597–615.

D. Benson, S. B. Iyengar, H. Krause, Stratifying modular representations of finite groups, Ann. of Math. 175 (2012)

D. Benson, S. B. Iyengar, H. Krause, Representations of finite groups: Local cohomology and support, Oberwolfach Seminar 43, Birkhäuser 2012.

D. Benson, S. B. Iyengar, H. Krause, Local cohomology and support for triangulated categories, Ann. Scient. Ec. Norm. Sup. (4) 41 (2008), 1–47.

D. Benson, S. B. Iyengar, H. Krause, Pevtsova, Stratification and π-cosupport: Finite groups, Math. Z. (to appear); arXiv:1505.06628

D. Benson, S. B. Iyengar, H. Krause, Pevtsova, Sratification for module categories of finite group schemes; arXiv:1510.06773

D. Benson, S. B. Iyengar, H. Krause, Pevtsova, Local duality for representations of finite group schemes; arXiv:1611.04197

W. Bruns and J. Herzog, Cohen–Macaulay rings, Cambridge Studies in Advanced Mathematics, vol. 39, Cambridge University Press, 1998.

A. I. Bondal, M. M. Kapranov, Representable functors, Serre functors, and mutations, Izv. Akad. Nauk SSSR Ser. Mat. 53 (1989), no. 6, 1183–1205, 1337; translation in Math. USSR-Izv. 35 (1990), no. 3, 519–541.

J. F. Carlson, The varieties and cohomology ring of a module, J. Algebra 85 (1983), 104–143.

E. M. Friedlander, J. Pevtsova, π-supports for modules for finite groups schemes, Duke Math. J. 139 (2007), 317–368.

P. Gabriel, Des catégories abéliennes, Bull. Math. Soc. France 90 (1962), 323—448

D. Happel, Triangulated categories in the representation theory of finite dimensional algebras, London Math. Soc. Lecture Note Series, vol. 119, Cambridge University Press, 1988

M. J. Hopkins, Global methods in homotopy theory, Homotopy Theory, Durham 1985, Lecture Notes in Mathematics, vol. 117, Cambridge University Press, 1987

M. Hovey, J. H. Palmieri, N. P. Strickland, Axiomatic stable homotopy theory, Mem. AMS, vol 128, American Math. Soc., 1997

S. Iyengar, G. Leuschke, A. Leykin, C. Miller, E. Miller, A. Singh, U. Walther, Twenty-four hours of local cohomology, Graduate Stud. Math. 87, American Mathematical Society, Providence, RI, 2007

H. Krause, A short proof of Auslander's defect formula, Linear Alg. Appl., 365 (2003), 267–270

E. Matlis, Injective modules over Noetherian rings, Pacific J. Math. 8 (1958), 511– 528

A. Neeman, The chromatic tower for D(R), Topology 31 (1992), 519—532

D. Quillen, The spectrum of an equivariant cohomology ring: I, II, Ann. Math. 94 (1971) 549-572, 573-602.