OPEN QUESTIONS

ANDY: $\Gamma_n(p) = \ker\left(SL_n(\mathbb{Z}) \longrightarrow SL_n(\mathbb{Z}/p)\right)$

QUESTION: $H^{vcd}(\Gamma_n(p), \mathbb{Q})$?

$$H^{vcd}(\Gamma_n(p), \mathbb{Q}) = \left(St_n(\mathbb{Z})\right)_{\Gamma_n(p)}$$

(DUALITY GROUP WITH SAME DUALIZING MODULE AS $SL_n(\mathbb{Z})$ AS FINITE INDEX)

KNOWN:

$$St_n(\mathbb{Z}) \twoheadrightarrow St_n(\mathbb{F}_p) = \mathbb{Q}^{p^{\binom{n}{2}}}$$

$\Gamma_n(p)$-INVARIANT

(FROM $T_n(\mathbb{Z}) \longrightarrow T_n(\mathbb{F}_p)$)

$\Rightarrow$ COINVARIANTS ARE AT LEAST $\mathbb{Q}^{p^{\binom{n}{2}}}$

ACTUALLY BIGGER:

$$\begin{tikzcd} T_n(\mathbb{Z}) \arrow[r] \arrow[dr] & T_n(\mathbb{F}_p) \\ & \left(T_n(\mathbb{Z})/\Gamma_n(p)\right) \arrow[u, "\text{NOT AN ISO!}"'] \end{tikzcd}$$

(BECAUSE MANY UNITS INSIDE $\mathbb{F}_p$ ... DET ISSUE)

KNOWN: THE MAP $\left(St_n \mathbb{Z}\right)_{\Gamma_n(p)} \twoheadrightarrow \tilde{H}_{n-2}\left(T_n(\mathbb{Z})/\Gamma_n(p)\right)$ IS SURJECTIVE

QUESTION: IS IT AN ISOMORPHISM?

(A GENERALIZATION OF ["ON THE DOUBLE COSET FORMULA" PHD THESIS OF [PARASCHIVESCU]

SANDER: SS $E^1_{pq} = H_q(GL_p(\mathbb{Z}), St_p(\mathbb{Q}) \otimes \Lambda)$

$\uparrow \mathbb{Z}[\frac{1}{7!}]$

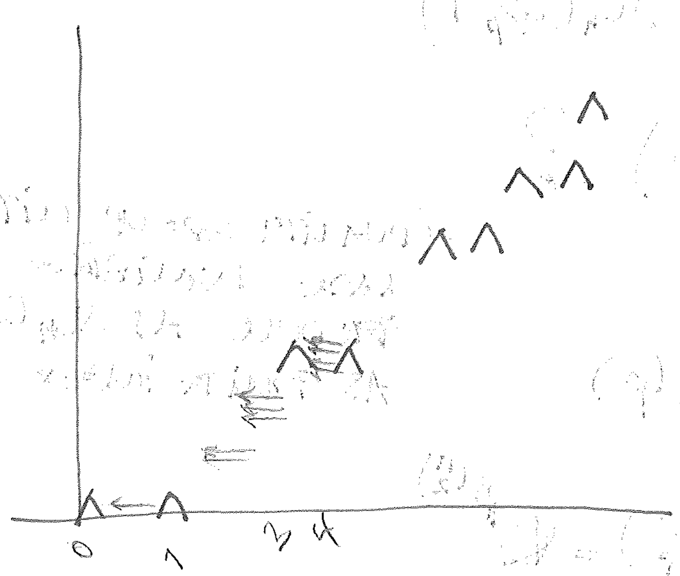
$\cong$

$H^*(GL_p(\mathbb{Z}))$

p is ODD

(OTHERWISE LOCAL SYSTEM)

BY DUALITY.



QUESTION: $H^*(GL_p(\mathbb{Z}), \mathbb{Q})$ ARE KNOWN FOR $* << p$ AND p is ODD APPEAR (HIGH) ON E^1-PAGE OF SS. WHAT HAPPENS TO THEM IN $H_{p+q}(\Omega^{\infty-1}\underline{K}(\mathbb{Z}), \mathbb{Q})$?

- MULTIPLICATIVE STRUCTURE OF SS?
- CAN ONE DEDUCE ~~WHERE~~ EXISTENCE OF UNSTABLE GL_n-HOMOLOGY OUT OF THE SS AND WHAT WE KNOW ABOUT K-THEORY?
- WRITE DOWN THE d_1-DIFFERENTIAL.

ANDY: $\mathcal{O}$ NUMBER RING $(\mathbb{Z}, \mathbb{Z}[i], \mathbb{Z}[\sqrt{2}], \dots)$ WITH FIELD OF FRACTIONS K.

QUESTION: WHAT IS $H^{vcd}(SL_n\mathcal{O}; \mathbb{Q})$?

$$\| \text{ (DUALITY)}$$

$$(St_n(K))_{SL_n(\mathcal{O})}$$

THM (CHURCH–FARB–PUTMAN)

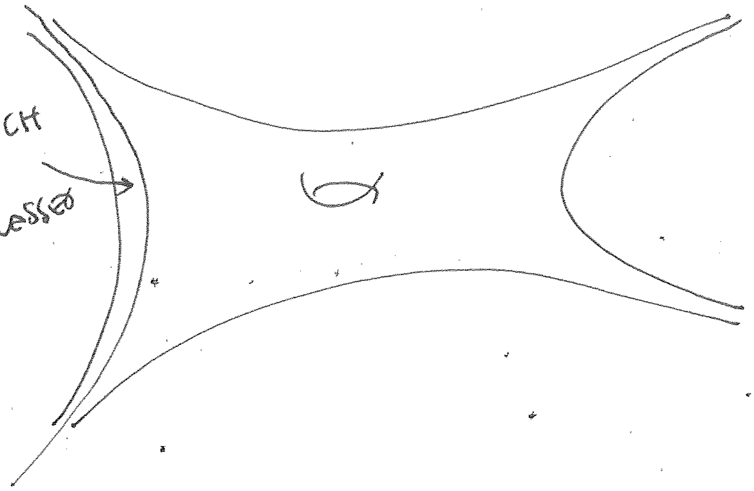
$$\dim H^{vcd}(Sl_n \mathcal{O}; \mathbb{Q}) \geq (cl\mathcal{O} - 1)^{n-1}$$

$cl\mathcal{O}$: CLASS NBR OF $\mathcal{O}$

$$= \dim\left(\tilde{H}_{n-2}\left(\mathcal{T}_n(k) / Sl_n\mathcal{O}; \mathbb{Q}\right)\right)$$

$n=2$ SYM. SPACE$_2$ $/ Sl_2\mathcal{O}$ = MFD (ORBIFOLD) WITH $cl\mathcal{O}$ CUSPS

USE SUCH LINES TO DETECT CLASSES



QUESTION: WHAT HAPPENS FOR SYMPLECTIC GROUPS?

- CONSTRUCT UNSTABLE COHOMOLOGY OF $Sl_n(\mathbb{Z})$
- GIVE STRUCTURAL RESULTS ABOUT $H^* Sl_n(\mathbb{Z})$

CONJECTURE (ATTRIBUTED TO SUSLIN) F INFINITE FIELD.

$$H_*(Gl_n(F), \mathbb{Q}) \hookrightarrow H_*(Gl_{n+1}(F), \mathbb{Q})$$ (ALL DEGREES!)

(STRONG INJECTIVITY CONJECTURE)

RANK CONJECTURE: $\mathrm{Im}(H_*(Gl_n(F), \mathbb{Q}) \to H_*(Gl_\infty(F), \mathbb{Q})) \cap \mathrm{Prim}(H_*(Gl_\infty(F), \mathbb{Q}))$

$$F_n^{rk} K_*(F) \otimes \mathbb{Q} :=$$

CONJ: $F^{\mathrm{rank}}_{\cdot} K_*(F) \otimes \mathbb{Q}$

COMPLEMENTARY.

$F^{\gamma}_{\cdot} K_*(F) \otimes \mathbb{Q}$

"γ FILTRATION"

— TRUE FOR NUMBER FIELDS BY BOREL – YANG.