Abstracts

Mini-courses

Akhil Mathew (Harvard University)

Nilpotence, Descent, and Algebraic K-theory

Let $A \to B$ be a Galois extension. One obtains a basic comparison map in algebraic K-theory $K(A) \to K(B)^{hG}$, between the K-theory of A and the homotopy fixed points of the G-action on the K-theory of B. This map is generally not an equivalence, but a fundamental theorem of Thomason implies that under suitable hypotheses the map becomes an equivalence after periodic localization. More recently, the growing field of structured ring spectra and Rognes's theory of Galois extensions has raised the question of finding analogs of these results when A and B are structured ring spectra.

The purpose of these lectures will be to describe recent joint work with Clausen, Naumann, and Noel on Galois and Galois-style descent in the algebraic K-theory of ring spectra, inspired by the classical work of Thomason.

There are several foundational ingredients that go into our result. In particular, we will begin with the theory of nilpotence and thick subcategories, starting from the work of Devinatz, Hopkins, and Smith. We will use a (proved) conjecture of May that the nilpotence of elements in the homotopy of E_∞-ring spectra can be checked after passage to ordinary homology. Another ingredient is the derived induction and restriction theory developed in joint work with Naumann and Noel, which assigns to a G-equivariant spectrum a "derived defect base" of subgroups from G. This leads to generalizations of Quillen-style stratification in complex oriented cohomology theories. Equivariant homotopy theory interacts with algebraic K-theory, and in particular the descent problem can be formulated in terms of G-spectra. We will finally put everything together and describe how one can prove several special cases of a Galois descent conjecture of Ausoni-Rognes and some variants.

Oscar Randal-Williams (University of Cambridge)

Moduli Spaces of Manifolds

The study of diffeomorphism groups of manifolds has recently been reinvigorated by developments leading on from Madsen–Weiss' proof of the Mumford conjecture. As the symmetries of smooth manifolds, these groups play a central role in differential topology and geometry, and trying to study them has led to new developments in homological stability, cobordism categories and TQFT's, and spaces of positive scalar curvature metrics. I will survey some of the techniques and latest developments surrounding this subject.

Nathalie Wahl (University of Copenhagen)

Homological Stability of Automorphism Groups

Many families of groups have by now been shown to display a stability in their homology. Examples include symmetric groups, braid groups, automorphisms of free groups, many linear groups, many mapping class groups and diffeomorphism groups. I'll explain what these examples have in common, and give a recipe for proving new stability results.

Talks

Monday

Felix Wierstra

Generalized Hopf Invariants

The classical Hopf invariant is an invariant of homotopy classes of maps $f : S^{4n-1} \to S^{2n}$, but can be generalized to arbitrary maps of two topological spaces X and Y. In general we can generalize the Hopf invariants even further to a more algebraic setting by combining the homotopy groups of X and the cohomology of Y. In this talk I shall give an introduction to these generalized Hopf invariants in rational homotopy theory and explain how the Hopf invariants can be generalized to other more algebraic settings, including characteristic p. This is joint work in progress with Dev Sinha.

Christoph Winges

On the Farrell-Jones Conjecture for Algebraic K-theory of Spaces

The Farrell-Jones conjecture predicts that the evaluation of certain homotopy invariant functors at the classifying spaces of discrete groups can be obtained via an induction-type formula. This talk will be concerned with the version of the conjecture dealing with Waldhausen's algebraic K-theory of spaces (aka A-theory). I will introduce the conjecture and present recent progress on the validity of this conjecture which adapts earlier work by Bartels, Lück, Reich, Wegner and others for the algebraic K-theory of group rings. If time permits, I will also outline how this relates to the study of automorphism groups of manifolds. This is joint work with Nils-Edvin Enkelmann, Wolfgang Lück, Malte Pieper and Mark Ullmann.

Will Mycroft

Plethories of Unstable Cohomology Operations

Cohomology theories assign algebraic invariants to topological spaces. The collection of operations of a cohomology theory has a very rich structure. I will discuss historic attempts to understand this structure, introduce the theory underlying a more modern approach to the problem, and illustrate some examples.

Malte Pieper

An Approach to Homotopy Groups of Automorphism Spaces

We give a (rough) overview on connections between the homotopy groups of automorphism groups of manifolds, surgery theory and algebraic K-theory, which in principle makes calculations of the former possible in various cases.

Piotr Pstragowski

The Underlying Homotopy Theory of Resolution Model Categories One of the classical problems in homotopy theory is to describe topological objects with prescribed algebraic invariants. Nowadays, one of the standard approaches is to define a moduli space of such objects and to decompose it as a well-understood limit of simpler spaces – this is for example the case in the Goerss-Hopkins obstruction theory to the existence of commutative ring spectra. The main technical tool in these obstruction theories are the so-called resolution (or E_2) model categories, I will give a short introduction to these ideas and show how using the recognition principle of Lurie one can with no difficulty identify the underlying homotopy theory of these model categories as that of product-preserving presheaves on the ∞-category of projective generators.

Anne Isabel Gaudreau

Almost Classical Knots and Their Surfaces

The story of knots as topological objects dates back to the 19th century. In the last few decades, it took a sharp turn with the introduction of virtual knots which allowed to generalise many classical tools to knots on surfaces up to stabilisation. The goal of this talk is to present various properties of virtual knots which are called almost classical, that can be defined as being those which are homologically trivial as knots on surfaces.

Dimitri Zaganidis

Homotopy Coherent Monad Morphisms

The aim of this talk is to present the (large) quasi-category of homotopy coherent monads in a sufficiently complete category $\mathcal{K}$ enriched in quasi-categories. Categories enriched in quasi-categories models $(\infty, 2)$-categories, so we will start by looking at the strict case, when $\mathcal{K}$ is a actually a 2-category. We define and study monads, adjunctions, and their morphisms in this context, and interpret them as 2-functors $\mathcal{D} \to \mathcal{K}$, where $\mathcal{D}$ is the universal 2-category containing a monad, an adjunction or a morphism. We replace $\mathcal{D}$ by $N_*\mathcal{D}$, the simplicial category obtained from $\mathcal{D}$ by taking the nerve of each of the hom-categories. Simplicial functors $N_*\mathcal{D} \to \mathcal{K}$ are the homotopy coherent versions of monads, adjunctions and their morphisms in $\mathcal{K}$. Through a nerve construction, we build a (large) simplicial set $N_{\mathrm{Mnd}}(\mathcal{K})$. When $\mathcal{K}$ admits the construction of algebras, we show that this simplicial set is indeed a quasi-category.

Renee Hoekzema

Manifolds with Odd Euler Characteristic

It is well-known that orientable manifolds can only have odd Euler characteristic if the dimension is a multiple of 4. For example, all orientable surfaces have even Euler characteristic, even though the non-orientable real projective plane has Euler characteristic 1. I will talk about my generalisation of this result for manifolds with "higher orientations". For example, spin manifolds can only have odd Euler characteristic if the dimension is a multiple of 8.

Hongyi Chu

Two Models for the Homotopy Theory of Infinity Operads

We compare two models for infinity operads: the complete Segal operads of Barwick and the complete dendroidal Segal spaces of Cisinski and Moerdijk. Combining this with comparison results already in the literature, this implies that all known models for infinity operads are equivalent. In particular, it follows that the homotopy theory of Lurie's infinity operads is equivalent to that of dendroidal sets and that of simplicial operads.

Jan-Bernhard Kordaß

Moduli Spaces of Riemannian Metrics with Curvature Bounds

It is well-known that the space of all complete Riemannian metrics over a smooth manifold is a contractible space on which the diffeomorphism group acts via pullback of metrics. Imposing geometric restrictions invariant under this action (e.g. curvature bounds) give rise to topologically more complicated moduli spaces. This expository talk will be concerned with various phenomena known about moduli spaces of metrics with lower curvature bounds over both closed and open manifolds.

Tuesday

Tom Sutton

Rational Homotopy Types of Exterior Algebras

A problem of classifying differential graded algebras (dgas) with a specified homology up to quasi-isomorphism is known as a "formality problem", and is a much studied area, over a variety of ground rings. As an example, for all primes p and $q \geq 0$, Dugger and Shipley have classified the homotopy types of connective dgas over $\mathbb{Z}$ with homology isomorphic to $\mathbb{F}_p(x_q)/(x_q^2)$, where x_q lives in degree q.

In my talk we will instead work over $\mathbb{Q}$, and all dgas will be commutative. One would hope that making such restrictions would simplify the problem, and allow one to be more ambitious in a different direction. Indeed, for a reasonable class of non complete intersection $\mathbb{Q}$-algebras k (concentrated in degree 0) I will present a very explicit computation of the "space of homotopy types" of connective cdgas over $\mathbb{Q}$ with homology isomorphic to $k[x_m]/(x_m^2)$, for any $m \geq 1$.

The main pieces of background covered in the talk will be a brief discussion of rational homotopy theory (in the senses of both Quillen and Sullivan), and the particular form of Andre-Quillen cohomology we use to classify extensions of k.

Surya Raghavendran

Introduction to the BV-formalism

The Batalin-Vilkovisky (BV) formalism gives a homological approach to defining the path integral in perturbative QFT. This (expository) talk will introduce the BV formalism via the example of 0-dimensional QFT and show that it recovers the usual combinatorics used by physicists in computing correlators.

Truls Raeder

Equivariant rational homotopy theory

I will begin by talking about the classical splitting of rational equivariant spectra indexed on a complete universe, as well as its effect on the corresponding Mackey functors. I will then go on to describe analogous results for commutative ring spectra, and the effect on the corresponding Tambara functors.

Kaj Börjeson

Free loop spaces and $\mathcal{A}_\infty$ algebras

The homology of free loop spaces is notoriously hard to compute. One approach is via Hochschild cohomology. If the space is formal and coformal we can use this together with Koszul duality to obtain a small tractable complex computing the free loop space homology. When the space is not formal we are naturally led to develop Koszul duality for $\mathcal{A}_\infty$-algebras. I will talk about how one can do this and how one can apply this in concrete computations.

Alice Hedenlund

Galois Theory of Structured Ring Spectra

Galois theory on ring spectra is a relatively new and fascinating area of algebraic topology. Introduced in the 2000's by John Rognes, it is inspired by the "brave new ring" paradigm and the hope is that it may play a similar part in the study of algebraic K-theory, as classical Galois theory plays for number theory. This is an expository talk on the subject in which we look at the necessary definitions, important examples and discuss the significance it may have in the further study of algebraic K-theory.

Megan Maguire

Stable and Unstable Homology of Configuration Spaces

In it's weakest form, we say that a family of topological spaces is homologically stable if for fixed i the ith homology groups of X_n and X_{n+1} are isomorphic for n sufficiently large. Notions of homological stability have been investigated for a wide range of topological families, including Hurwitz spaces, moduli of curves, and configuration spaces. Arnol'd first proved integral homological stability for the unordered configuration spaces of $\mathbb{R}^2$. This was extended to open (connected, orientable, finite type) manifolds by McDuff and Segal (independently), and recently Church (via the method of representation stability), followed by Randal-Williams (via a method more akin to Segal), proved rational homological stability for all (connected, orientable, finite type) manifolds. Using the tools of Totaro, we compute the Betti numbers, both stable and unstable, of the unordered configuration spaces of some example spaces, such as a genus one Riemann surface and $\mathbb{CP}^3$, and prove a vanishing theorem about the unstable homology a la Church, Farb, and Putman (joint with Melanie Wood).

Calvin Woo

Iwasawa Theory and Algebraic K-theory

This will be an expository talk, starting by introducing how results in Iwasawa theory can be adapted to computing the algebraic K-theory of number rings. We'll trace a thread of thought from Thomason to Dwyer-Mitchell relating homotopy types of certain K-theory spectra in terms of etale cohomological objects. In the end, we'll survey recent work of Blumberg-Mandell and others that point towards an Iwasawa theory for highly structured ring spectra.

Martin Palmer

Homological Stability for Symmetric Diffeomorphism Groups of Manifolds and Diffeomorphism Groups of Manifolds with Singularities

I will talk about a homological stability result for moduli spaces of submanifolds of a fixed ambient manifold, where the stability occurs with respect to the number of (mutually isotopic) components of the submanifold. This generalises the classical result of McDuff and Segal for configuration spaces of points (the 0-dimensional case) and is similar in spirit to recent results of Cantero-Randal-Williams and Kupers, concerning connected subsurfaces and unlinks in $\mathbb{R}^3$ respectively. As a corollary, one may deduce homological stability for so-called "symmetric diffeomorphism groups" of manifolds with respect to the operation of connected sum along a submanifold - generalising a recent result of Tillmann which is concerned with connected sum at a point (the usual connected sum). A special case of this is homological stability for the diffeomorphism groups of certain manifolds with Baas-Sullivan singularities with fixed singularity type. This connects to work of Perlmutter on the cobordism categories of such objects, and thereby suggests a conjectural description of the stable homology of these diffeomorphism groups.

Juan Villeta-Garcia

Stabilizing Spectral Functors of Exact Categories

Algebraic K-Theory is often thought of as “the” universal additive invariant of rings (or more generally, exact categories). Often, however, functors on exact categories don't satisfy additivity. We will describe a procedure (due to McCarthy) that constructs a functor's universal additive approximation. We will then apply it to different local coefficient systems, recovering known invariants of rings (K-Theory, THH, etc.). We will talk about what happens when we push these coefficient systems to the world of spectra, and tie in work of Lindenstrauss and McCarthy on the Taylor tower of Algebraic K-Theory.

Matthias Grey

On Rational Homological Stability for Automorphisms of Manifolds

In this talk we discuss rational homological stability results for the classifying spaces of the homotopy automorphisms and block diffeomorphisms of iterated connected sums of products of spheres. The stability result for the homotopy automorphisms follows from the homological stability of its fundamental group with certain twisted coefficients. The stability for the block diffeomorphisms is concluded by using surgery theory.

Wednesday

Jens Jakob Kjær

The Homology of Algebras over the Spectral Lie Operad

One of the big classical results of algebraic topology is the computation of the homology operations on infinite loop spaces done by Kudo and Araki, and Dyer and Lashof. These Dyer-Lashof operations were seen by May, by recognizing the infinite loop space as an algebra over an E_∞-operad, as coming from operadic data. From Goodwillie calculus we have the notion of the derivatives of the identity functor on topological spaces, and Ching has shown that this has the structure of an operad, called the Spectral Lie Operad. A natural question is therefore to study algebras over this operad, as these also have some importance to functor calculus. Behrens showed that with $\mathbb{Z}/2$-coefficients we have Dyer-Lashof-like operations, as well as a "Lie bracket", and a complete description of these was given by Antolin-Camarena. I will discuss the analogue for odd primes. I will begin this talk with reminding people of the definition of operads as well as the construction of the Dyer-Lashof operations and certain key facts pertaining to them. I will then go on to describe the construction of the Dyer-Lasof-like operations and the "Lie bracket", as well as sketching how to prove the relations these satisfy.

Yumi Boote

Symmetric Squares of Even Spaces

The integral homology of the symmetric square of a finite connected CW complex X has been known since the 1960's, although the general answer is very complicated. However, the situation for the integral cohomology ring remains an open problem, except for a few special cases. One of the main difficulties is the computation of the ring structure. In this talk I shall outline a solution for even X, which have torsion free integral cohomology, concentrated in even dimensions. Examples include quaternionic projective spaces and the octonionic projective plane, for which the results may be confirmed by independent geometric methods.

Thursday

Sune Precht Reeh

Representation Rings for Fusion Systems and Dimension Functions

Given a representation V of a finite group G we can associate a dimension function that to each subgroup H of G assigns the dimension of the fixed point space V^H. The dimension functions are "super class functions" that are constant on the conjugacy classes of subgroups in G. For a p-group the list of Borel-Smith conditions characterizes the super class functions that come from real representations. In a joint paper with Ergün Yalcin we study the representation rings for saturated fusion systems using characteristic bisets and idempotents, and we give a list of Borel-Smith conditions that characterize the dimension functions of the real representations that are stable under the fusion system. Finally, we give applications of these results to the problem of constructing homotopy G-spheres for a finite group G with prime power isotropy and (a multiple of) a predetermined dimension function.

Inbar Klang

Factorization Homology of Thom Spectra

By a result of Lewis, the Thom spectrum of an E_n-algebra map to BO is an E_n-ring spectrum. We give a description of the factorization homology of Thom spectra that arise in this way, as Thom spectra over a certain mapping space or section space. This can be viewed a form of "twisted non-abelian Poincaré duality" and can be used to calculate the factorization homology of some interesting Thom spectra.

David Sprehn

Modular Characteristic Classes for Representations over Finite Fields

In my joint work with Anssi Lahtinen, we have constructed a new system of characteristic classes in mod-p cohomology for representations (or, equivalently, vector bundles) over the finite field of order p^r. These vanish on decomposable representations. The construction of the classes is explicit and simple. We are able to evaluate them on some representations of elementary abelian p-groups, and thereby show that many of them are nonzero. This means that there are new families of nonzero classes in the modular cohomology of the general linear groups over finite fields, about which very little is known to date. In particular, the only previously known nonzero elements (except for small n) occur in degrees at least exponential in n; we produce nonzero classes in degrees linear in n. For much of the talk, I will restrict to the field of order 2, as the results there are simpler.

Ben Knudsen

Configuration Spaces and Higher Enveloping Algebras

A theorem of Milnor and Moore asserts that, under mild hypotheses, any cocommutative Hopf algebra is the universal enveloping algebra of a Lie algebra. In this talk, we will encounter a Hopf-like structure on the disjoint union of the configuration spaces of Euclidean space; we stack configurations side by side to multiply, and we split them apart to comultiply. Interpreting configuration spaces as a kind of enveloping algebra leads to a wealth of computations and a new proof of homological stability.

Anders Husebø

Homotopy Fixed Points of Iterated Topological Hochschild Homology

In 2013 Veen calculated in his thesis that the self map of connective Morava K-theory of iterated topological Hochschild homology (THH) of finite fields is non-zero, supporting the conjecture that homotopy fixed points of iterated THH increases telescopic complexity. We would like to also be able to say that the above map is an isomorphism in high degrees by using the same techniques, but for this we need a spectral sequence in which we can calculate more terms. One way of doing this is taking into account that the general linear group of the integers also acts on iterated THH so we could look at these homotopy fixed points. We will introduce the Loday functor for spectra which when evaluated on tori give iterated THH, then look at some spectral sequence for how to calculate homotopy fixed points. Then we end up with calculating some classical invariant theory, especially in the case of two iterations giving some further evidence for the redshift conjecture.

Nina Friedrich

Homological Stability of Moduli Spaces of High Dimensional Manifolds

We first explain how to translate homological stability in the geometric setting of moduli spaces of high dimensional manifolds to the algebraic setting of quadratic forms. For simply-connected manifolds, Galatius and Randal-Williams have shown that certain simplicial complexes arising on the algebraic side are highly connected, and hence deduced homological stability theorems for moduli spaces of simply-connected manifolds. We generalise this to a much larger class of manifolds (those having virtually polycyclic fundamental group).

Arpon Raksit

Characters in Global Equivariant Homotopy Theory

The classical theory of characters from the representation theory of finite groups can be rephrased as a comparison between equivariant K-theory and ordinary cohomology. Hopkins-Kuhn-Ravenel gave a generalization of this, providing a comparison between any (Borel-)equivariant Morava E-theory and ordinary cohomology. I'll try to explain this story, and how it fits into the framework of global equivariant homotopy theory.

William Gollinger

Signatures of elements of homotopy groups of Madsen-Tillmann spectra

The homotopy groups of the Madsen-Tillmann spectrum $MTSO(d)$ have an interpretation as bordism groups of oriented manifolds whose tangent bundles stably reduce to a rank d bundle; when the dimension of the manifold is divisible by 4 it has a potentially interesting signature. This talk will adapt a divisibility result from the (unstable) vector field problem to this case, and provide some examples of elements of these groups which attain the minimal positive signature.

Martina Rovelli

A Looping-Delooping Adjunction for Topological Spaces

Every principal G-bundle is classified up to equivalence by a homotopy class of maps into the classifying space of G. On the other hand, for every nice topological space Milnor constructed a strict model of the loop space, that is a group. Moreover the morphisms of topological groups defined on the loop space of X generate all the bundles over X up to equivalence. We will show that the relationship between Milnor's loop space and the classifying space functor is, in a precise sense, an adjoint pair between based spaces and topological groups in a homotopical context. This proof leads to a classification of principal bundles with a fixed structure group. Such a result clarifies the deep relation that exists between the theory of bundles, the classifying space construction and the loop space construction.

Thomas Wasserman

Topological Gauge Theory

Topological gauge theories are topological quantum field theories that also take into account the data of principal G-bundles on the ingredients of the bordism category. In this talk we will discuss such theories where G is a finite group and the bordism category in question is the bicategory of one-dimensional manifords, two-dimensional cobordisms and three-dimensional cobordisms between them. The aim of this talk is to give an overview of once-extended three-dimensional field theories in general and topological gauge theory in particular, and to advertise work in progress towards classifying the latter.

Friday

Catherine Ray

Polynomial Ham Sandwich theorem

Gromov proved that given r finite volume subsets of R^n, there exists a degree $d(n, r)$-hypersurface bisecting these sets, as long as r is less than $(n+d$ choose $n)-1$. We describe a fun and elementary proof of this theorem, and come to understand this mysterious bound!

Magdalena Zielenkiewicz

Five Perspectives on the Exceptional Group G_2

I will introduce the exceptional group G_2 presenting various perspectives - topological, algebraic, combinatorial, physical and analytic. I will present an easy combinatorial description of the equivariant cohomology ring of G_2.