ABSTRACTS FOR MASTER CLASS IN
SUMS OF SELF-ADJOINT OPERATORS:
KASPAROV PRODUCTS AND APPLICATIONS
UNIVERSITY OF COPENHAGEN 22-26/8–2016

Francesca Arici (Radboud University Nijmegen),
Mapping cone exact sequences for Cuntz-Pimsner algebras
Mapping cones have played a central role in studying the properties of KK-theory, and have likewise been used to further the study of non-commutative topology and dynamics. In this talk we will elaborate on the relationship between Cuntz-Pimsner algebras and mapping cone algebras. We will describe an explicit isomorphism between the Cuntz-Pimsner exact sequence and the mapping cone exact sequence in K-theory for the inclusion of the coefficient algebra into the Pimsner algebra, for a class of finite-index bimodules satisfying some additional conditions. Possible applications include to the computation of K-theory (and K-homology) groups of C^*-algebras of locally finite graphs.

Chris Bourne (University of Erlangen-Nuremberg),
Disordered topological states and Kasparov theory
Disordered systems in solid state physics can be modelled by a discrete or continuous crossed product C^*-algebra, with the physical invariants of interest arising as real or complex index pairings. In this talk, we will briefly review the achievements and current limitations of applying index theory and KK-theory to solid state systems. This is joint work with Alan Carey, Johannes Kellendonk and Adam Rennie.

Jens Kaad (University of Southern Denmark),
Descent and index pairings in weighted KK-theory
In this talk I will introduce a bivariant functor which associates an abelian group to a pair consisting of a dense subalgebra of a C^*-algebra and a C^*-algebra both equipped with an action of a discrete group. This bivariant functor, called weighted equivariant KK-theory, resembles the KK-theory of a pair of C^*-algebras but it incorporates an extra feature: It is possible to "weigh" the C^*-modules by a generalized length operator for the group. This makes the weighted KK-theory more related to Lafforgue's KK-theory for Banach algebras. We shall then investigate the descent properties of weighted KK-theory, and we shall show that weighted KK-theory provides index homomorphisms between K-groups of C^*-algebras and not just their respective dense subalgebras.

Johannes Kellendonk (University Lyon I),
Introduction to topological insulators
I will explain the physical aspects of topological insulators and where K-theory, and its dual, come into play.

Matthias Lesch (University of Bonn),
Sums of regular self-adjoint operators
My aim is to cover the content of two relatively recent (2012, 2013) papers jointly written with Jens Kaad; both papers concern regular operators in Hilbert C^*-modules.

Hilbert C^*-modules are the analogues of Hilbert spaces where the C^*-algebra plays the role of the scalar field. They enter prominently in Kasparov's celebrated KK-theory, which is by now a standard tool in operators algebras.

While the elementary properties of Hilbert C^*-modules can be derived basically in parallel to Hilbert space theory, the lack of an analogue of the Projection Theorem leads to obstructions and interesting new phenomena. In particular the theory of unbounded operators is notoriously more complicated due to the additional axiom of regularity.

Unbounded operators are unavoidable whenever one deals with geometrically defined differential operators, as such operators never act as bounded operators in a Hilbert space/module.

My plan is to give a leisurely introduction to the topics; the audience is expected to have some background in functional analysis and operators algebras, no specific knowledge of Hilbert module theory is expected.

The three talks are roughly broken down as follows:

Part I (First and part of second talk): Gentle introduction to unbounded operators in Hilbert C^* modules, spectral theory, subtle difference between symmetric and self-adjoint operators, regularity problem, examples of nonregular selfadjoint operators; Explanation of the Local Global Principle for regularity. Proof of the local global principle, discussion of corner cases.

Part II (remainder of second talk, third talk): Construction of unbounded representatives of the Kasparov product; application to the Spectral Flow Theorem and index theory for Dirac Schrödinger operators. Open problems. Depending on the background of the audience, a crash course on Dirac operators might be included.

Bram Mesland (University of Hannover),
Frames and connections in KK-theory
A vital ingredient in the construction of the unbounded Kasparov product is the so-called connection operator. Elaborating on some of the more technical aspects of Rennie's lectures, in these talks I will show how the notion of a frame in a Hilbert C^*-module gives a natural way to construct the connection operator. Subsequently I will address the problem of the self-adjointness of this operator and explain a set of sufficient conditions to obtain self-adjointness.

Fedor Sukochev (University of New South Wales),
Introduction to double operator Integration and quantum differentiability of essentially bounded functions on euclidean space
Given an essentially bounded function f on $\mathbb{R}^d$ with $d > 1$ we prove a formula for the Dixmier trace of the d:th power of the absolute value of the commutator of pointwise multiplication by f with a Riesz transform. This result is motivated by Connes' quantised calculus and allows us to give necessary and sufficient conditions for the commutator to lie in the weak trace class ideal $\mathscr{L}_{d,\infty}$. This talk gives an overview of Connes' theory of infinitesimals and the methods of singular traces and double operator integrals which are used in the proof.

Adam Rennie (University of Wollongong),
Applications of KK-theory in non-commutative geometry and physics
I will give a brief introduction to the Kasparov groups and product. This will be illustrated by examples and recent applications. The examples will aim for concreteness, and the applications include the bulk-edge correspondence for the quantum Hall effect.

Koen van den Dungen (SISSA, Trieste),
Locally bounded perturbations and (odd) unbounded KK-theory
Assuming the existence of a suitable approximate identity, we will show that 'locally bounded' perturbations of regular self-adjoint operators are again regular and self-adjoint. As a consequence, we will see that unbounded Kasparov modules are stable under locally bounded perturbations. A typical example is a perturbation of an unbounded Kasparov module by an unbounded multiplier of the algebra. As a final application, we will see that any (even) unbounded representative of an odd KK-theory class naturally yields an odd unbounded Kasparov module representing the same class.

Walter van Suijlekom (Radboud University Nijmegen),
Factorizations in KK-theory and applications to physics
We give an overview of the literature on factorizations of spectral triples in KK-theory, explicitly written in terms of the unbounded (internal) Kasparov product. We discuss applications to gauge theory in physics, including Yang-Mills gauge theory and the celebrated Standard Model of particle physics.