

# Symposium: The mathematical legacy of Uffe Haagerup

June 24–26, 2016

*Titles and abstracts*

**Dietmar Bisch:** *Composition of Subfactors*

Group-type subfactors are the simplest examples of composed subfactors that were extensively studied by Haagerup and myself. Other, more general compositions were later developed by Jones and myself, and our constructions continue to be a rich source of examples of “exotic” infinite depth subfactors. I will present several results that grew out of the idea of composing subfactors.

**Joachim Cuntz:** *Left regular  $C^*$ -algebras for semigroups from number theory*

We describe the left regular  $C^*$ -algebras associated with semigroups arising from the ring of algebraic integers in a number field. This includes a determination of their K-theory as well as of their KMS-states.

**Ken Dykema:** *Haagerup–Schultz subspaces and applications*

Uffe’s many profound contributions include a result, obtained with Hanne Schultz and published in 2009, about the existence of hyperinvariant subspaces for operators in  $\text{II}_1$ -factors, that effect decompositions of the operators according to Brown measure. This talk will focus on some ideas around this result and Uffe and Hanne’s (marvellous) proof, as well as, time permitting, some applications.

**Ed Effros:** *The triumph of abstraction*

Owing to the work of such individuals as Grothendieck and Connes, the basic architecture of mathematics has been fundamentally transformed. Perhaps the most important insight is the unexpected unity of the subject. For example, notions from algebraic geometry have profound implications for functional analysis and vice versa, and in turn these are closely related to ideas in modern physics. We will examine a particular aspect of this phenomenon in the “classification theory” for unitary group representations. In his attempt to generalize the results from the purely algebraic theory of finite group representations to second countable locally compact groups, Mackey discovered a fundamental dichotomy for the classification spaces (the “duals”) of (separable) irreducible unitary representations. He conjectured that the well-behaved (“smooth”) duals corresponded exactly to the “type I”

global structure of non irreducible unitary representations. This was subsequently proved by Glimm and Jaffe, one of the key results of abstract analysis in the last century. Similar results have been generalized to a wide range mathematical objects. We will briefly describe a remarkable paper of Ando, Haagerup, and Winsløw who use the space of von Neumann algebras to address the famous Connes' QWEP conjecture.

**Vaughan Jones:** *Haagerup's work in subfactors*

In the mid 1990s Uffe Haagerup caused a sensation in the subfactor world by constructing a subfactor of index  $\frac{5+\sqrt{13}}{2}$  and showing it is the (finite depth) subfactor of smallest index larger than 4. It is now known simply as “the Haagerup”. This began a large research effort trying to understand Haagerup's ideas and extend them to as large a value of the index as possible. There have been two independent new constructions of the Haagerup but it is fair to say that it remains mysterious.

I will explain some of Uffe's ideas and attempt to survey the progress that they have inspired.

**Richard V. Kadison:** *The Meaning of Unitary Operators*

We discuss Uffe's role in the study of convex combination of unitary operators in  $C^*$ -algebras — especially in the difficult “boundary question” raised by C. Olsen and G. K. Pedersen and Uffe's solution.

**Narutaka Ozawa:** *Furstenberg boundary and  $C^*$ -simplicity*

I will talk about the recent results of Kalantar–Kennedy, Breuillard–Kalantar–Kennedy–Ozawa, Kennedy, and Haagerup which characterize those discrete groups whose reduced group  $C^*$ -algebra is simple.

**Gilles Pisier:** *Sidon sets in duals of compact groups and generalizations*

We will recall some of the classical theory of Sidon sets of characters on compact groups (Abelian or not). We will then give several recent extensions to Sidon sets, randomly Sidon sets and subgaussian sequences in bounded orthonormal systems, following recent work by Bourgain and Lewko, and by the author, both currently available on arxiv. The case of matricial systems, analogous to Fourier-Peter-Weyl series on compact groups, connects the subject to random matrix theory. An unpublished result of Rider (circa 1975) will also be highlighted.

**Sorin Popa:** *On the 1-cohomology of  $II_1$  factors*

One of the most important problems in  $II_1$  factors is to find a “good” 1-cohomology theory, i.e., one that is non-vanishing (and if possible calculable) and that could detect important properties of a  $II_1$  factor  $M$ , like absence of regularity, or infinite generation. I will comment on two attempts in this direction: one concerning derivations of  $M$  into  $B(L^2M)$  satisfying some smoothness properties (Johnson-Parrott 1974, Popa 1984, Galatan-Popa 2014); and one concerning derivations of  $M$  into  $\text{Aff}(M \otimes M)$  (Connes-Shlyakhtenko 2004, Thom 2007, Alekseev-Kyed 2012-2014, Popa-Vaes 2014).

**Dimitri Shlyakhtenko:** *Free entropy dimension and the first  $L^2$  Betti number*

In this talk we discuss some old and new results on the connection between Voiculescu’s free entropy dimension and the first  $L^2$  Betti number of a discrete group. In particular, we prove that if  $G$  is a discrete finitely generated finitely presented group with vanishing first  $L^2$  Betti number, then its von Neumann algebra cannot be isomorphic to a non-amenable free product.

**Roland Speicher:** *Linearization and Brown measure*

With his investigations on the Brown measure and the linearization philosophy Uffe introduced new tools in free probability theory and started exciting new directions in the subject. Much of my work in the last decade is inspired by those ideas. I will report on some of those results.

**Andreas Thom:** *Groups of finite type*

An example of a SIN and unitarily representable polish group is provided which is not a closed subgroup of the unitary group of a finite von Neumann algebra. This answers a question of Sorin Popa. Our construction relies on a new perspective towards uniformly bounded representations and combines the theory of operators affiliated with a finite von Neumann algebra with various Banach algebraic techniques such as the Maurey-Nikishin factorization theorem.

**Stefaan Vaes:** *Structure and classification of free Araki-Woods factors*

Free Araki-Woods factors are a free probability analog of the type III hyperfinite factors. They were introduced by Shlyakhtenko in 1996, who completely classified the free Araki-Woods factors associated with almost periodic orthogonal representations of the real numbers. I present a joint work with Houdayer and Shlyakhtenko in which we completely classify a large class of non almost periodic free Araki-Woods factors. The key technical result is a deformation/rigidity criterion for the unitary conjugacy of two faithful normal states on a von Neumann algebra. I also discuss several structural properties of free Araki-Woods factors, including their strong solidity from a joint work with Boutonnet and Houdayer.

**Alain Valette:** *A survey of weak amenability*

Weak amenability was introduced in a famous paper by Cowling and Haagerup (1989). We will survey recent developments of the subject, emphasizing the parallel with the close but inequivalent Haagerup property (or a-T-menability). We will present Haagerup's proof (written in 1986, published in 2016) that  $SL(3, \mathbb{R})$  is not weakly amenable.

**Anatoly Vershik:** *Theory of filtrations, standardness and AF-algebras*

1. Classification of dyadic filtrations, Examples and RWRS.
2. Standardness of the filtrations and “highest” 0-1 Laws.
3. Standard Bratelli diagrams and AF-algebras.

**Dan Voiculescu:** *Commutants mod Normed Ideals*

The talk will be about operator algebras which are commutants mod normed ideals of operators on Hilbert space. This will include connections with entropy,  $K$ -theory aspects and properties of coronas.