Representation theory of groups, quantum groups, and operator algebras

University of Copenhagen

1–5 June 2015

Minicourses

Siegfried Echterhoff

Title: Crossed products, the Mackey-Rieffel-Green machine and applications

Lecture 1) Induced representations of groups in the sense of Mackey and Blattner. Induced representations of C*-algebras via Hilbert modules in the sense of Rieffel. The connection between both of them.
Lecture 2) Group extensions and twisted crossed products. The imprimitivity theorem.
Lecture 3) The orbit method for crossed products and group extensions.
Lecture 4) Twisted group algebras and the little group method.

Nigel Higson

Title: C^*-algebras and the noncommutative geometry of real reductive groups

The irreducible and usually infinite-dimensional representations of real reductive groups (think SL(n,R)) have been studied and classified by Harish-Chandra, Langlands and friends. The standard approach taken by operator algebraists—to construct a convolution algebra on the group, then study irreducible representations of the algebra—has usually not been the route followed in representation theory. Nevertheless there are some interesting possibilities in this direction, as I'll try to indicate in these lectures.

Christian Voigt

Title: Complex semisimple quantum groups and representation theory

In these lectures I will give an introduction to the theory of complex semisimple quantum groups. These quantum groups are deformations of classical complex semisimple Lie groups, and an instructive special case is the quantum Lorentz group, which is obtained by deforming the group $SL(2,\mathbb{C})$ in a suitable way. After giving some general background on quantum groups and their representations, the first aim will be to describe the construction of the C^*-algebras associated with complex quantum groups. I will then focus on some aspects of their representation theory, indicating similarities/differences to the situation for classical complex groups, and illustrating the results in the case of the quantum Lorentz group. Time permitting, I will explain how this relates to K-theory and the Baum-Connes conjecture.

Research Talks

Yuki Arano

Title: Unitary spherical representations of Drinfeld doubles

It is known that the Drinfeld double of a q-deformation of a compact Lie group can be seen as a quantization of the complexification of the original compact Lie group. In this talk, we investigate the unitary representation theory of such Drinfeld double via its analogy to that of the complex Lie group and classify the unitary spherical representations in the case of $SU_q(3)$. As an application, we show central property (T) for $SU_q(2n+1)$.

Ingrid Beltita

Title: Fourier transforms on C^*-algebras of nilpotent Lie groups

We will discuss the image of operator valued Fourier transforms on C^*-algebras of general nilpotent Lie groups. We use the method of coadjoint orbits and appropriate stratifications, due to N.V. Pedersen, of the duals of nilpotent Lie algebras. We thus show that the C^*-algebra of any nilpotent Lie group is a solvable C^*-algebra, that is, it admits a finite increasing sequence of closed two-sided ideals whose successive quotients are isomorphic to algebras of compact-operator valued continuous functions that vanish at infinity on suitable locally compact spaces. Also we show that it is isomorphic to a C^*-algebra of compact-operator valued piecewise continuous functions that satisfy certain boundary conditions at the boundary of layers in the spectrum.
In addition, a characterization of Heisenberg groups in terms of group C^*-algebras will be provided along these lines. The talk is based on joint work with Daniel Beltita and Jean Ludwig.

Kenny De Commer

Title: Partial compact quantum groups

T. Hayashi introduced the notion of 'compact quantum group of face type', which is to be seen as a compact quantum groupoid with a finite object set. In this talk, we introduce the notion of 'partial compact quantum group', which is a generalization of Hayashi's definition to the case of an infinite object set. Partial compact quantum groups can for example be constructed from any rigid tensor C*-category, and from any ergodic action of a compact quantum group. We give some details on a concrete example related to the dynamical quantum SU(2) group. This is joint work with T. Timmermann.

Pierre Julg

Title: Baum-Connes conjecture and property T

Kazhdan's property T makes life difficult for those who try to prove the Baum-Connes property. We shall review different approaches to overcome the difficulty. The use of Jolissaint rapid decrease functions (V. Lafforgue 1998) has given the first examples of proof of the Baum-Connes conjecture (without coefficient) for some (very) specific groups. Concerning the general conjecture (with coefficients), the idea of using uniformly bounded or slowly increasing representations (N. Higson, PJ, V. Lafforgue) is adapted to the case of Gromov hyperbolic groups (V. Lafforgue 2012) or to the rank one simple Lie groups (my work in progress).

Sergey Neshveyev

Title: Representation theory for monoidal categories

Given a rigid C^*-tensor category, I'll explain how to define a C^*-completion of the fusion algebra of the category analogous to the construction of the full group C^*-algebra of a discrete group. The representation theory of this C^*-algebra is related to the Drinfeld center of the original category, or more precisely, of its ind-completion. The C^*-algebra allows one to define various approximation properties of monoidal categories such as property (T). I will explain what these constructions mean in case of quantum groups and subfactors. (Joint work with Makoto Yamashita.)

Roger Plymen

Title: An Introduction to the local Langlands correspondence: the example of SL_n

This talk will be about the smooth irreducible representations of the special linear group $SL_n(F)$ where F is a local non-archimedean field, for example $F = \mathbb{Q}_p$. The local Langlands correspondence provides parameters for these representations via arithmetic data, notably Galois groups. The emphasis in this talk will be on L-packets and Vogan packets. The group SL_2 will hopefully illustrate some of the ideas.

Eyal Subag

Title: Algebraic Families of Harish Chandra pairs and their modules

In my talk I will try to convince you that Lie groups come in natural algebraic families. A construction of such families that relates different real forms of GL(n,C), and SL(n,C) will be given. Moreover, we shall see that we can naturally associate families of Harish Chandra pairs to these families of groups. For the family that goes through SU(2), SU(1,1), and their Cartan motion group, a classification of generically irreducible Harish Chandra modules will be given. As an application, a formulation of the Mackey bijection between the duals of SU(1,1) and its Cartan motion group in terms of families of Harish Chandra modules will be presented. The talk is based on joint works with Joseph Bernstein and Nigel Higson.
