

WORKSHOP ON AUTOMORPHISMS OF FREE GROUPS
CENTRE FOR SYMMETRY AND DEFORMATION
UNIVERSITY OF COPENHAGEN
9–13 NOVEMBER 2015

Schedule

| | Monday | Tuesday | Wednesday | Thursday | Friday |
|-------|-----------------|-------------------|-----------------|------------------|----------------|
| 9:00 | Registration | | | | |
| 9:30 | Vogtmann | Sakasai | Vogtmann | Vespa | Griffin |
| 10:30 | Coffee | Coffee | Coffee | Coffee | Coffee |
| 11:00 | Conant | Suzuki | Kassabov | Djament | Szymik |
| 12:00 | Lunch | Lunch | Lunch | Lunch | Lunch |
| 13:00 | | | | | Putman |
| 13:45 | | Berglund | | | |
| 14:30 | Satoh | | Lahtinen | Bartholdi | Cake |
| 14:45 | | Cake | | | |
| 15:15 | | <i>Colloquium</i> | | | |
| 15:30 | Cake | | Cake | Cake | |
| 17:30 | Pizzas | | | | |
| 18:00 | | | Dinner | | |

All talks will be in **Auditorium 8** or **Auditorium 10** (see below for details).

Coffee, cake and pizzas are served in the common room **04.4.19**.

The workshop dinner on Wednesday is at restaurant Bindia, Blegdamsvej 130.

There is a department colloquium on Tuesday in Auditorium 4.

Monday November 9th

09:00 - 09:30. Registration

09:30 - 10:30, Aud. 8. **Karen Vogtmann:** Assembling homology classes in automorphism groups of free groups.

This talk will be an introduction to the recent paper by Conant, Hatcher, Kassabov and myself. After brief historical background I will define the groups and spaces we study and the action of the symmetric group on their homology. These spaces and groups are related by “assembly maps”, which allow us to bootstrap information about the homology of small rank free groups to get new information about larger rank groups. I will end with some conjectures suggested by this work.

10:30 - 11:00. Coffee

11:00 - 12:00, Aud. 8. **Jim Conant:** The Lie Lie algebra.

The homology of $\text{Out}(F_n)$, by work of Kontsevich, is closely tied to the homology of a Lie algebra associated with the Lie operad. This Lie algebra is also the target of the Johnson homomorphism for surfaces with at least one boundary component. The abelianization of this Lie algebra is important for constructing homology classes for $\text{Out}(F_n)$ and for understanding the cokernel of the Johnson homomorphism. Work of Conant-Kassabov-Vogtmann[CKV] showed the abelianization has a natural grading by homotopy type of certain graphs. The degree 1 part was known to Morita, detected using his trace map. The higher degree parts were shown by CKV to correspond to certain twisted cohomology groups of $\text{Out}(F_n)$, and the degree 2 case was calculated. In this talk, we will discuss progress on the degree 3 case and how it relates both to $\text{Out}(F_n)$ and the Johnson cokernel.

12:00 - 14:30. Lunch break

14:30 - 15:30, Aud. 10. **Takao Satoh:** On the Johnson homomorphisms of automorphism groups of free groups.

First, we recall some facts about the Johnson homomorphisms and twisted first cohomology groups of the automorphism groups of free groups. Then we introduce the rings of $SL(2, \mathbb{C})$ -representations of free groups. By using it, we can construct analogs of the Johnson homomorphisms.

In this talk, we show that the first homomorphism can be extended to the automorphism group of a free group as a crossed homomorphism, and show that it induces Kawazumi’s 1-cocycle and Morita’s 1-cocycle of the automorphism group of a free group.

15:30 Cake

17:30 Pizzas

Tuesday November 10th

09:30 - 10:30, Aud. 10. **Takuya Sakasai**: Structure of symplectic invariant Lie subalgebras associated with symplectic derivation Lie algebras.

We discuss structures of the symplectic invariant tensors in the Lie algebra of symplectic derivations of the free Lie algebra generated by the fundamental symplectic representation. We describe the orthogonal direct sum decomposition of the Lie algebra induced by the canonical metric on it. By using this, we determine the structure of the Lie algebra completely up to degree 6 including the unstable cases where the genus 1 case has an independent meaning. This is a joint work with Shigeyuki Morita and Masaaki Suzuki.

10:30 - 11:00. Coffee

11:00 - 12:00, Aud. 10. **Masaaki Suzuki**: Integral Euler characteristic of the outer automorphism groups of free groups.

We compute the integral Euler characteristic of the outer automorphism groups of free groups of rank up to 11 by using Kontsevich's theorem and representation theory. This is a joint work with Shigeyuki Morita and Takuya Sakasai.

12:00 - 13:45. Lunch break

13:45 - 14:45, Aud. 10. **Alexander Berglund**: Automorphisms of high dimensional manifolds and free groups.

There is a classical programme for understanding diffeomorphisms of high dimensional manifolds whereby one studies, in turn, the monoid of homotopy automorphisms, the block diffeomorphism group, and finally the diffeomorphism group. The difference in each step is measured by, respectively, the surgery exact sequence and, in a range, Waldhausen's algebraic K -theory of spaces.

Recently, we established homological stability and calculated the rational stable cohomology of the homotopy automorphisms and the block diffeomorphisms of the g -fold connected sum $\#^g S^d \times S^d$ minus a disk ($2d > 4$). The stable cohomology of the homotopy automorphisms turns out to be expressible in terms of Kontsevich's graph complex associated to the Lie operad, which is known to compute the rational homology of $\text{Out}(F_n)$. Even more surprisingly, the stable cohomology of the block diffeomorphisms is expressible in terms of a graph complex similar to the "hairy graph complex" introduced by Conant-Kassabov-Vogtmann, whose homology is related to the homology of the groups $\Gamma_{n,s} = F_n^{s-1} \rtimes \text{Aut}(F_n)$. I will also comment on the relation to the work of Galatius and Randal-Williams on the stable cohomology of the diffeomorphism group; there are natural guesses for what graph homology classes correspond to the generalized Miller-Morita-Mumford classes. This is joint work with Ib Madsen.

14:45 Cake

15:15 *Department colloquium*: Jack Copeland, *The Mathematical Objection: Turing, Gödel, and the Penrose of the Mind*.

Wednesday November 11th

9:30 - 10:30, Aud. 10. **Karen Vogtmann**: Assembling homology classes in automorphism groups of free groups, part II.

(continuation of the talk on Monday)

10:30 - 11:00. Coffee

11:00 - 12:00, Aud. 10. **Martin Kassabov**: Hopf algebras and representation varieties.

I will describe representations of $\text{Aut}(F_n)$ and $\text{Out}(F_n)$ arising from (co)commutative Hopf algebras. Even though this construction does not lead to new representations of $\text{Aut}(F_n)$ it gives a uniform way to consider all known finite dimensional representations of these groups. The idea behind this construction leads to a new viewpoint of the representation variety of a discrete group Γ in a Lie group G . I will speculate how this construction can be modified to obtain quantization of the representation varieties. One of the aims of this project is to obtain a new description for quantum representations of mapping class groups. (Based on joint works with J. Conant and A. Patotski.)

12:00 - 14:30. Lunch break

14:30 - 15:30, Aud. 10. **Anssi Lahtinen**: Automorphism groups of free groups and string topology.

In recent joint work with Richard Hepworth, I have extended string topology of classifying spaces of compact Lie groups into a novel kind of field theory incorporating operations parametrized by homology groups of automorphism groups of free groups with boundaries. In this talk, I will discuss computations of higher string topology operations in the context of this theory, and explain how these computations lead to nontrivial elements in the ordinary mod 2 homology of holomorphs of free groups and in the twisted homology of automorphism groups of free groups.

15:30 Cake

18:00 Dinner

Thursday November 12th

9:30 - 10:30, Aud. 8. **Christine Vespa**: Stable homology of automorphism groups of free groups and functor homology (I).

Functor homology (i.e. homological algebra in functor categories) on a suitable category allows us to compute some stable homology of automorphism groups. For automorphism groups of free groups, stable homology with coefficients in a reduced covariant polynomial functor (e.g. tensor power or exterior power of the abelianisation) is trivial. For coefficients given by a reduced contravariant polynomial functor the stable homology is no more trivial. Djament (see his talk in this workshop) proved recently that it is given by homology of functors from groups to abelian groups. In this talk, I will give several results concerning this homology of functors.

10:30 - 11:00. Coffee

11:00 - 12:00, Aud. 8. **Aurélien Djament**: Stable homology of automorphism groups of free groups and functor homology (II).

We will explain how to prove that the stable cohomology of automorphism groups of free groups with coefficients in the exterior algebra of their abelianisation is, rationally, an explicit symmetric algebra, proving a conjecture of O. Randal-Williams. Our main tool is functor homology, involving homological comparison (through derived Kan extensions) of many different categories of free groups. We also need Galatius rational cancellation of stable reduced homology of automorphism groups of free groups (with constant coefficients) and some Ext-computations in functor homology due to C. Vespa (see her talk in this workshop).

12:00 - 14:30. Lunch break

14:30 - 15:30, Aud. 8. **Laurent Bartholdi**: Free group automorphisms and Lie algebra.

A construction due to Magnus associates a Lie algebra with any discrete group, by considering the associated graded to a descending central series, such as the lower normal series.

There is some hope, starting by work of Andreadakis in 1963, to understand the automorphism group of a free group using its associated Lie algebra. This Lie algebra is naturally a subalgebra of the algebra of “free differential operators”, and I will explain which part of that Lie algebra describes the automorphism group of a free group. In passing, I will answer some open questions by Andreadakis and Bryant.

15:30 Cake

Friday November 13th

09:30 - 10:30, Aud. 8. **James Griffin:** The homology of $\text{Aut}(H * G * \dots * G)$.

I will define a spectral sequence computing the homology of the automorphism group of a given free product of groups. The first page splits as a coend of a functor of moduli spaces of derivative labelled graphs of fixed rank r and a functor of tensor powers of the chain complexes of the indecomposable free factors and their automorphism groups. This factorises the homology of the automorphism group into a part depending on the rank r of the free group factor and a part depending on the other free factors. We will then study the moduli space of derivative labelled graphs by using known results about the homology of relative automorphism groups of a free group.

Three consequences of this are: 1) homological stability for $\text{Aut}(H * G * \dots * G)$ follows from homological stability for relative automorphism groups of free groups; 2) a Lie-graph homology for automorphisms of a given free product; and 3) a conjectural description of the stable homology of $\text{Aut}(H * G * \dots * G)$.

10:30 - 11:00. Coffee

11:00 - 12:00, Aud. 8. **Markus Szymik:** On the homology of automorphism groups of algebraic structures related to free groups.

Universal algebra provides for a context in which families of automorphism groups naturally appear, in which homological stability is often satisfied, and where the stable homology can sometimes be computed. This talk will be an introduction to this point of view and present recent work of mine on examples that are closely related to the theory of groups.

12:00 - 13:00. Lunch break

13:00 - 14:00, Aud. 8. **Andrew Putman:** The high-dimensional cohomology of the moduli space of curves with level structures.

I will prove that the moduli space of curves with level structures has an enormous amount of rational cohomology in its cohomological dimension. This is joint work with Neil Fullarton.

14:30. Cake

List of Participants

| Name | Affiliation |
|------------------------------|--|
| Espen Auset Nielsen | University of Copenhagen |
| Laurent Bartholdi | University of Göttingen |
| Alexander Berglund | Stockholm University |
| Clarisson Rizzie Canlubo | University of Copenhagen |
| Federico Cantero | Université catholique de Louvain |
| Christopher Cave | University of Copenhagen |
| Jim Conant | University of Tennessee |
| Jacques Darné | Lille 1 University |
| Dieter Degrijse | University of Copenhagen |
| Aurélien Djament | University of Nantes |
| Daniela Egas Santander | Freie Universität Berlin |
| Nina Friedrich | University of Cambridge |
| Mauricio Gomez Lopez | University of Southern Denmark |
| Matthias Grey | University of Copenhagen |
| James Griffin | University of Coventry |
| Jesper Grodal | University of Copenhagen |
| Ryo Horiuchi | University of Copenhagen |
| Martin Kassabov | Cornell University |
| Manuel Krannich | University of Copenhagen |
| Anssi Lahtinen | University of Hamburg |
| Kang Li | University of Copenhagen |
| Daniel Lütgehetmann | Freie Universität Berlin |
| Ib Madsen | University of Copenhagen |
| Francisco Mbiavanga Da Silva | Agostinho Neto University |
| Sam Nariman | University of Münster |
| Martin Palmer | University of Paris 13 |
| Irakli Patchkoria | University of Copenhagen |
| Peter Patzt | Freie Universität Berlin |
| Dan Petersen | University of Copenhagen |
| Tomasz Prytuła | University of Copenhagen |
| Andrew Putman | Rice University |
| Deividas Sabonis | Technical University Munich |
| Takuya Sakasai | University of Tokyo |
| Takao Satoh | Tokyo University of Science |
| David Schrittmesser | University of Copenhagen |
| Arthur Soulié | Université de Strasbourg |
| David Sprehn | University of Copenhagen |
| Masaaki Suzuki | University of Meiji |
| Markus Szymik | NTNU Trondheim |
| Massimiliano Ungheretti | University of Copenhagen |
| Olga Varghese | University of Münster |
| Christine Vespa | University of Strasbourg |
| Karen Vogtmann | Cornell University/University of Warwick |
| Nathalie Wahl | University of Copenhagen |
| Guozhen Wang | University of Copenhagen |