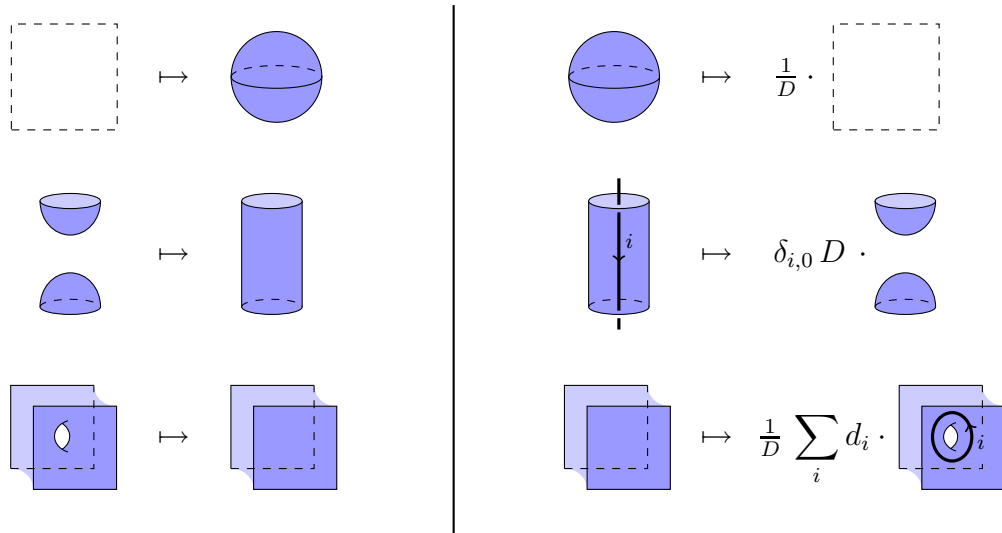


## RESHETIKHIN-TURAEV INVARIANTS

Let  $M$  be a 3-manifold that bounds a 4-manifold  $W$ . After replacing  $W$  by a cobordant manifold,  $\exists$  a submersion  $f : W \rightarrow \mathbb{R}$  s.t.  $f|_M$  is Morse and each  $f^{-1}(t)$  is a union of handlebodies;  $f$  has 6 types of critical points (the pictures below). To compute the invariant  $Z(M) = Z(M, W)$ , watch the movie  $\{f^{-1}(t)\}_{t \in \mathbb{R}}$  ( $t$  is time) and do the operations below to the internal string diagrams. At the end of the movie, you have your number:



Here,  $d_i = \bigcirc^i$  and  $D^2 = \sum_i d_i^2$ .

**Example:** the RT invariant of the 3-torus:

$$\begin{aligned} & \text{Dashed square} \mapsto \text{Sphere} \mapsto \frac{1}{D} \sum_i d_i \cdot \bigcirc^i \mapsto \frac{1}{D} \sum_i d_i \cdot \text{Genus-2 surface with } i \text{ windings} = \\ & \frac{1}{D} \sum_i \text{Genus-2 surface with } i \text{ windings} + \text{terms of the form } j \neq 0 \mapsto \sum_i \text{Circle with } i \text{ windings} \mapsto \sum_i \text{Genus-2 surface with } i \text{ windings} = \\ & \sum_i \frac{1}{d_i} \text{Genus-2 surface with } i \text{ windings} + \text{terms of the form } j \neq 0 \mapsto D \sum_i \frac{1}{d_i} \cdot \bigcirc^i \mapsto D \sum_i \frac{1}{d_i} \cdot \text{Sphere} \\ & = D \sum_i \text{Sphere} \mapsto \sum_i 1 \cdot \text{Dashed square} = \#\{\text{simple objects}\} \end{aligned}$$