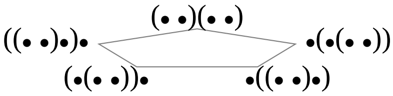
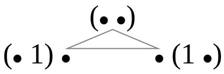
Categories

A category is **monoidal** (one also says **tensor**, when in a linear context) if it has a product functor $X, Y \mapsto X \otimes Y$, an associator $\alpha : (X \otimes Y) \otimes Z \cong X \otimes (Y \otimes Z)$ natural in X, Y, Z making the pentagon

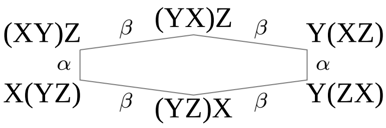


commute, a unit object 1, and natural isomorphisms $X \otimes 1 \cong X \cong 1 \otimes X$ making the triangle



commute.

A category is **braided** if it is monoidal and has a braiding $\beta : X \otimes Y \cong Y \otimes X$, natural in X and Y, making the hexagon



commute, and also the version of that axiom with β^{-1} in place of β.

A category is **symmetric monoidal** if it is braided and $\beta^2 = 1$.

A monoidal category is **rigid** if every object X has left & right duals. A left dual is: $(X^*, e : X^* \otimes X \to 1, c : 1 \to X \otimes X^*)$ s.t. $(1 \otimes e) \circ (c \otimes 1) = 1$, $(e \otimes 1) \circ (1 \otimes c) = 1$. Right duals are defined similarly.
Note that being rigid is just a property.

A tensor category is **fusion** if it is rigid, semisimple, and has finitely many types of simple objects.

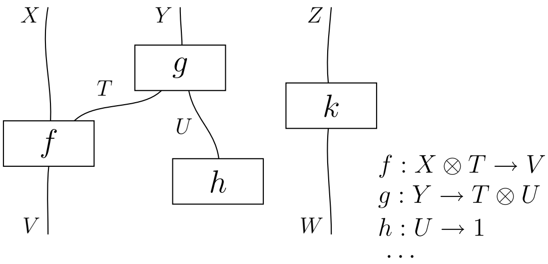
A category is **balanced** if it is braided and has twists $\theta_X : X \cong X$, natural in X, satisfying $\theta_1 = 1$ and $\theta_{X \otimes Y} = (\theta_X \otimes \theta_Y) \circ \beta^2$.

A category is **ribbon** if it is balanced, rigid, and $ev \circ (\theta \otimes 1) = ev \circ (1 \otimes \theta)$.

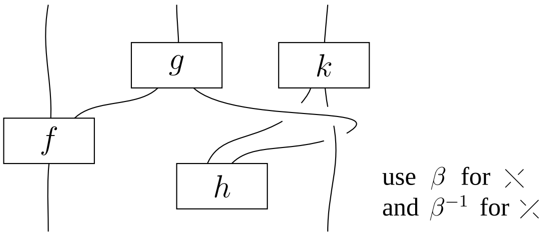
A category is **modular** if it is ribbon and the S-matrix $[\text{Hopf link}(i,j)]_{ij}$ is invertible.

String Diagrams

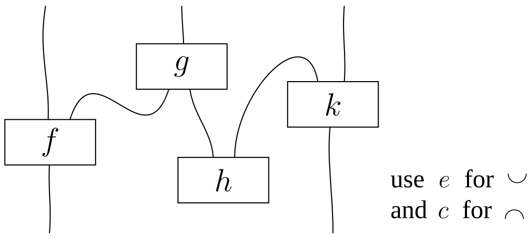
Monoidal. planar, strands only go down:



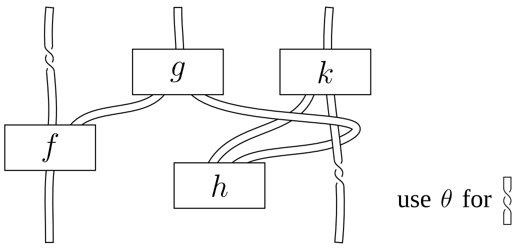
Braided. things are now in 3dim, strands go down, coupons not allowed to rotate:



Rigid. planar, strands may bend up and down, coupons not allowed to rotate:



Balanced. ribbons instead of strands, only down, coupons may rotate around z-axis:



Ribbon. ribbons in 3-space, all 3-dimensional isotopies are now allowed: