RYSZARD 3

STANDING ASSUMPTION: N, M HYPERFINITE II_1-FACTORS, $N' \cap M = \mathbb{C}1$, FINITE INDEX $N \subset M$ $(\dim_N L^2(M) < \infty)$, FINITE DEPTH

$\rightsquigarrow N \subset M = M_0 \subset M_1 \subset M_2 \subset \dots$

$M_1 = \operatorname{End} L^2(M_0)_N \quad \dots$

(IN PARTICULAR, M_1 MORITA EQUIV TO N) (GIVEN BY M_0 $\rightsquigarrow$ "PERIODIC SEQUENCE OF MORITA EQUIV")

RELATIVE COMMUTANTS:

$N' \cap M_0 \subset N' \cap M_1 \subset N' \cap M_2 \subset \dots$

$N' \cap M_0 = \mathbb{C}$

is a SEQUENCE OF fd $*$-ALGEBRAS $/\mathbb{C}$ (i.e. FINITE SUM OF MATRIX ALG'S)

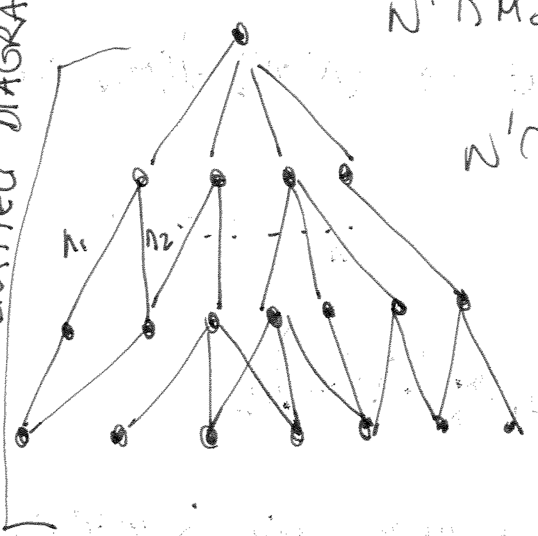
$A = \bigoplus_{i=1}^{n} M_{k_i}(\mathbb{C}) \sim \underbrace{\cdot \quad \cdot \quad \cdot \quad \cdot \quad \dots \quad \cdot}_{n}$

MINIMAL PROJECTION CORRESPONDING TO ith SUMMAND
CENTRE

$Z(A) = \bigoplus_{i=1}^{n} \mathbb{C}$

(i.e. A POINT CORRESPOND TO A SUMMAND OR TO THE COMPONENT OF THE CENTER CORRESP. TO THAT SUMMAND)

BRATTELI DIAGRAM



$N' \cap M_0$

$N' \cap M_1$

$N' \cap M_2 \dots$

CAN DESCRIBE SUCH MAPS BY A GRAPH WITH NUMBERS GIVING THE MULTIPLICITY

$M_n(\mathbb{C}) \longrightarrow M_k(\mathbb{C})$

HAS TO HAVE THE FORM

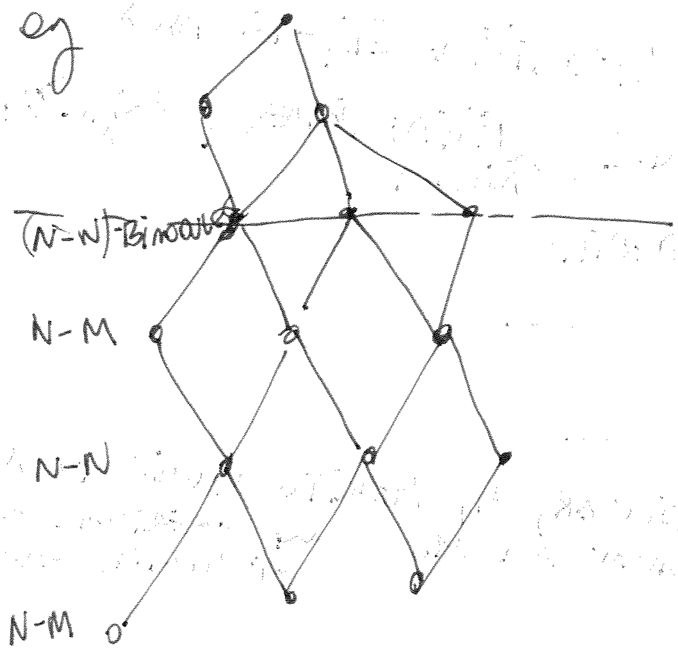
$$x \longmapsto \begin{pmatrix} x & 0 & 0 \\ 0 & x & 0 \\ 0 & 0 & x \end{pmatrix}$$

ONLY POSSIBLE IF $k = n\ell$

MULTIPLICITY

FINITE DEPTH
BECAUSE OF ~~[illegible]~~, THE PATTERN IN THE DIAGRAM WILL REPEAT ITSELF:

eg



THE DOTS ALSO REPRESENT IRREDUCIBLE BIMODULES

$$ {}_N L^2(M_0)_N = \bigoplus_i {}_N X^i_N \otimes m_i $$

MULTIPLICITY

IRREDUCIBLE N-N-BIMODULES

$$ \rightsquigarrow \ \mathrm{End}_N L^2(M_0)_N = \bigoplus_i M_{m_i}(\mathbb{C}) $$

(AS EACH X^i HAS NO ENDOMORPHISM)

$$ M_2 = \mathrm{End}\, L^2(M_1)_{M_0} \rightsquigarrow N' \cap M_2 = \mathrm{End}_N L^2(M_1)_{M_0} $$

ALSO HAVE A RESTRICTION $(N\text{-}M_0)$-BIMODULES $\to$ $(N\text{-}N)$-BIMODULES

$$ {}_N L^2(M_0)_N \xrightarrow{\text{INDUCE}} {}_N L^2(M_0) \otimes_N L^2(M_0)_{M_0} \xrightarrow{\text{RESTRICT}} {}_N L^2(M_1)_{N_1} \to {}_N L^2(M_1)_N \otimes_N L^2(M_0)_{M_0} \to \cdots $$

PROCESS THAT PRODUCES THE BIMODULES → THEY WILL REPEAT, BUT NEW ARE CREATED.

— WE GENERATE $(N-N)$-BIMODULES BY $-\underset{N}{\otimes} L^2(M_0)_N$

$(N-M)$-BIMODULES BY $-\underset{N}{\otimes} L^2(M_0)_{M_0}$

EDGES IN THE DIAGRAM CORRESPOND TO INDUCTION.

FINITE DEPTH $\Longleftrightarrow$ FINITELY MANY IRREDUCIBLE BIMODULES OCCUR

$\rightarrow$ THE GRAPH MUST REPEAT ITSELF AFTER SOME STEPS.

PRINCIPAL GRAPH = SUBGRAPH OF NEW MODULES

FIRST OCCURANCE.

HAVE ALSO THE CONDITIONAL EXPECTATION:

$$N \overset{E_0}{\longleftarrow} M_0 \overset{E_1}{\longleftarrow} M_1 \longleftarrow \dots$$

$$N \subset M_0 \subset M_1 \subset \dots$$

RECALL

M, $N \subset M$ $\rightsquigarrow$ $M \xrightarrow{E} N$ [STEINSPRING TOMIGANA 70's]

$L^2(N) \overset{e}{\subset} L^2(M)$

PROJECTION TO THE CLOSED SUBSPACE $L^2(N)$

THM: $eMe \simeq Ne$

$E(m)e = eme$

$N \ni n \longmapsto ne \in Ne = eMe$ iso

OTHER POINT OF VIEW: ON $M_1 \to M_0$

$M_1 = M_0 \underset{N}{\otimes} M_0$ $\quad (m \otimes m')(x \otimes x') = m\langle m' | x\rangle_N \underset{N}{\otimes} x'$

THEN $e = 1 \underset{N}{\otimes} 1$, $e^2 = e$