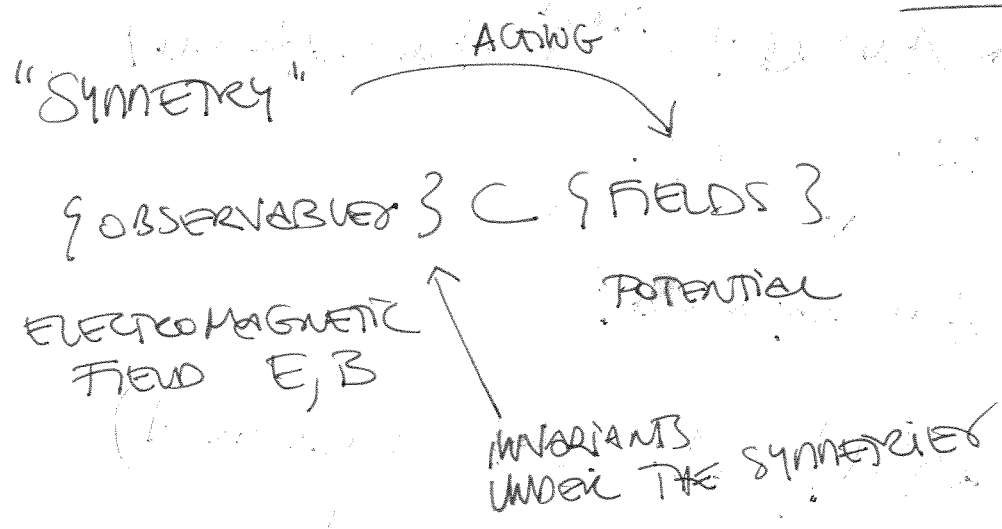


RYSZARD NEST 1

FROM SUBFACTORS TO INVARIANTS OF 3-MFDS



N VN-ALG $\hookrightarrow M$ VN-ALG (SOON WILL RESTRICT TO II₁-FACTORS)

IDEA: THE INTERPLAY BETWEEN N AND M SHOULD ALLOW TO RECOVER THE SYMMETRIES -- (SOME KIND OF SYMMETRIES)

GROUP, QUANTUM GROUP, ...

EX: N VN-ALG, G FINITE GROUP, $G \rightarrow \text{Out}(N)$

$N \times G = N[G] = M_0$ GROUP RING

$N \subset M_0$ $N \backslash M_0 = \mathbb{C}I$

QUESTION: CAN WE RECOGNIZE G ? FROM THE INCLUSION

$$G \text{ GROUP} \rightsquigarrow (C(G), \Delta)$$

COASS, COPRODUCT

(+ PRODUCT = PRODUCT OF FUNCTIONS)

COACTION OF G ON AN ALGEBRA A:

$$\Leftrightarrow A \xrightarrow{\delta} A \otimes C(G) \xrightarrow{id \otimes \Delta} A \otimes C(G) \otimes C(G)$$



$$A \otimes C(G) \otimes C(G)$$

EX: G FINITE. $(C(G), \Delta)$, $\Delta: C(G) \rightarrow C(G \times G)$

$$\Delta f(g, h) := f(gh)$$

SUPPOSE $G \curvearrowright \text{Aut}(A)$

$$\rightsquigarrow \delta_\alpha: A \rightarrow A \otimes C(G)$$

$$a \mapsto \alpha_g^{-1}(a)$$

DUAL: $(C^*(G), \hat{\Delta})$

LINEAR DUAL

COPRODUCT DUAL TO THE PRODUCT OF $C(G)$

(PRODUCT = CONVOLUTION)

EX: $C^*(G) \equiv$ GROUP ALGEBRA $\mathbb{C}[G]$

$$\delta_g \leftrightarrow g$$

"DELTA-FUNCTION AT g"

$$\hat{\Delta}: \delta_g \mapsto \delta_g \otimes \delta_g$$

$$\text{So } G \rightsquigarrow (C(G), m, \Delta)$$

$$(C^*(G), \hat{m}, \hat{\Delta})$$

HAVE THAT $C(G)$ ACTS ON $C^*(G)$: $C(G) \xrightarrow{\Delta} C(G) \otimes C(G)$
(AND COACTS ON ITSELF)

Action of $C^*(G) \iff$ COACTION OF $C(G)$



FOR A HOPF ALGEBRA, THIS IS A DEFINITION OF THE ACTION OF THE HOPF ALG ON SOMETHING, NAMED BY A COACTION OF THE DUAL COALGEBRA.

$$A \xrightarrow{\delta} A \otimes C(G)$$

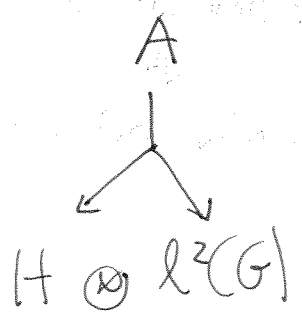
A IN ALG ACTS ON H

HILBERT SPACE

$$H \otimes \ell^2(G)$$

G, C(G) ACTS ON $\ell^2(G)$

G (QUANTUM) GROUP



$$A \rtimes G := \delta(A)(1 \otimes C^*(G))$$

(ACTS ON $H \otimes \ell^2(G)$)

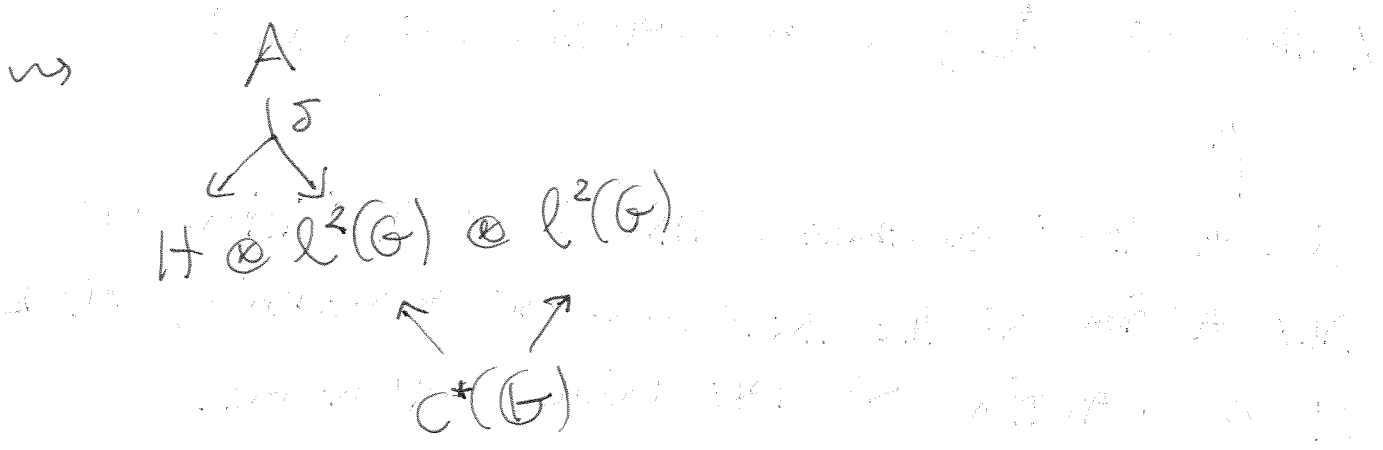
\exists UNITARY $W \subset H \otimes \ell^2(G) \otimes \ell^2(G)$

COMPUTING WITH THE ACTION OF A ON THE FIRST TWO FACTORS AND "IMPLEMENT" THE COPRODUCT ON THE $C^*(G)$ ACTION (?)

$$\ell^2(G) \otimes \ell^2(G) = \ell^2(G, \ell^2(G))$$

$W: g \rightarrow \underset{\substack{\text{multiplication} \\ \text{by } g}}{\lambda_g}$

$$(Wf)(g, h) = f(g, g^{-1}h)$$



$$A \rtimes G \longrightarrow A \rtimes G \otimes C^*(G) \quad \text{COACTION}$$

EX: G ABELIAN $C(G) = C^*(\hat{G})$
GROUP OF CHARACTERS ABELIAN

$$\begin{array}{l}
 A \rtimes G = \sum a_g \cdot g \quad \chi \in \hat{G} \\
 \sum a_g \cdot g \longrightarrow \sum a_g \chi(g) g
 \end{array}
 \quad \left| \begin{array}{l}
 \text{COACTION OF } C^*(\hat{G}) \\
 \updownarrow \\
 \text{ACTION OF } \hat{G}
 \end{array} \right.$$

$$N \subset N \rtimes G = M_0 \subset M_0 \rtimes \hat{G} = M_1 \subset M_1 \rtimes G = M_2 \subset \dots$$

SEQUENCE OF ALGEBRAS

BECAUSE G ACTS BY OUTER-AUTOMORPHISMS, $N' \cap M_0 = \mathbb{C}1$

THEN $N' \cap M_1 = C^*(\hat{G})$ COMMUTANT OF N INSIDE M_0

$$N' \cap M_2 = \overline{C(G) C^*(G)} = B(\ell^2(G))$$

$$M_0' \cap M_2 = C^*(G)$$

\longrightarrow GOT $C^*(\hat{G})$ AND $C^*(G)$ OUT OF THE SEQUENCE