RYSZARD NEST 1

FROM SUBFACTORS TO INVARIANTS OF 3-MFDS

"SYMMETRY" $\xrightarrow{\text{ACTING}}$

{OBSERVABLES} $\subset$ {FIELDS}

POTENTIAL

ELECTROMAGNETIC FIELD E, B

INVARIANTS UNDER THE SYMMETRIES

- N vN-ALG $\hookrightarrow M$ vN-ALG (SOON WILL RESTRICT TO II_1-FACTORS)

IDEA: THE INTERPLAY BETWEEN N AND M SHOULD ALLOW TO RECOVER THE SYMMETRIES -- (SOME KIND OF SYMMETRIES)
GROUP, QUANTUM GROUP, ...

EX: N vN-ALG, G FINITE GROUP, $G \to \mathrm{Out}(N)$

$$\rightarrow N \rtimes G = N[G] = M_0 \qquad \text{GROUP RING}$$

$$N \hookrightarrow N \rtimes G \qquad N' \cap M_0 = \mathbb{C}I$$

QUESTION: CAN WE RECOGNIZE G? FROM THE INCLUSION

G GROUP $\rightsquigarrow$ $(C(G), \Delta)$
└ COASS. COPRODUCT
(+ PRODUCT = PRODUCT OF FUNCTIONS)

<u>COACTION OF G</u> ON AN ALGEBRA A:

$$\Leftrightarrow \begin{tikzcd} A \arrow[r, "\delta"] & A \otimes C(G) \arrow[r, "id \otimes \Delta"] \arrow[d, "\delta \otimes id"'] \arrow[dr, phantom, "\circlearrowright"] & A \otimes C(G) \otimes C(G) \arrow[dl, equal] \\ & A \otimes C(G) \otimes C(G) & \end{tikzcd}$$

<u>EX</u>: G FINITE. $(C(G), \Delta)$, $\Delta : C(G) \to C(G \times G)$

$$\Delta f(g,h) := f(gh).$$

SUPPOSE $G \xrightarrow{\alpha} Aut(A)$

$\rightsquigarrow \delta_\alpha : A \longrightarrow A \otimes C(G)$

$a \longmapsto \alpha_g^{-1}(a)$

<u>DUAL</u>: $(C^*(G), \hat{\Delta})$

$C^*(G)$: LINEAR DUAL

$\hat{\Delta}$: COPRODUCT DUAL TO THE PRODUCT OF $C(G)$

(PRODUCT = CONVOLUTION)

<u>EX</u>: $C^*(G) \equiv$ GROUP ALGEBRA $\mathbb{C}[G]$

$\delta_g \leftrightarrow g$

"DELTA-FUNCTION AT g"

$\hat{\Delta} : \delta_g \longmapsto \delta_g \otimes \delta_g$

SO $G \rightsquigarrow (C(G), m, \Delta)$

$(C^*(G), \hat{m}, \hat{\Delta})$

HAVE THAT $C(G)$ ACTS ON $C^*(G)$: $C(G) \xrightarrow{\Delta} C(G) \otimes C(G)$

(AND COACTS ON ITSELF)

ACTION OF $C^*(G)$ $\iff$ COACTION OF $C(G)$

$\uparrow$

FOR A HOPF ALGEBRA, THIS IS A DEFINITION OF THE ACTION OF THE HOPF ALG ON SOMETHING, NAMELY BY A COACTION OF THE DUAL COALGEBRA.

$$A \xrightarrow{\delta} A \otimes C(G)$$

A VN ALG ACTS ON H HILBERT SPACE

$$H \otimes \ell^2(G)$$

G, $C(G)$ ACTS ON $\ell^2(G)$

G (QUANTUM) GROUP

$$\begin{array}{c} A \\ \downarrow \\ H \otimes \ell^2(G) \\ \uparrow \\ C^*(G) \end{array}$$

$$A \rtimes G := \delta(A)(1 \otimes C^*(G))$$

(ACTS ON $H \otimes \ell^2(G)$)

$\exists$ UNITARY $W \curvearrowright H \otimes \ell^2(G) \otimes \ell^2(G)$

COMMUTING WITH THE ACTION OF A ON THE FIRST TWO FACTORS AND "IMPLEMENTS THE COPRODUCT" ON THE $C^*(G)$ ACTION (?)

$$\ell^2(G) \otimes \ell^2(G) = \ell^2(G, \ell^2(G)), \quad W: g \longrightarrow \lambda_g$$

λ_g = MULTIPLICATION BY g

$$(Wf)(g,h) = f(g, g^{-1}h)$$

$\leadsto$

$$\begin{tikzcd} & A \arrow[dl] \arrow[d, "\delta"] & \\ H \otimes \ell^2(G) & \otimes & \ell^2(G) \\ & C^*(G) \arrow[ul] \arrow[ur] & \end{tikzcd}$$

$A \rtimes G \longrightarrow A \rtimes G \otimes C^*(G)$ COACTION

~~EX~~: G ABELIAN $C(G) = C^*(\hat{G})$

GROUP OF CHARACTERS
ABELIAN

$A \rtimes G = \sum a_g \cdot g$ $\chi \in \hat{G}$

$\sum a_g \cdot g \longrightarrow \sum a_g \chi(g)\, g$

COACTION OF $C^*(\hat{G})$
$\Updownarrow$
ACTION OF $\hat{G}$

$$N \subset N \rtimes G = M_0 \subset M_0 \rtimes \hat{G} = M_1 \subset M_1 \rtimes G = M_2 \subset \cdots$$

SEQUENCE OF ALGEBRAS

BECAUSE G ACTS BY OUTER AUTOMORPHISMS, $\underbrace{N' \cap M_0}_{\text{COMMUTANT OF } N \text{ INSIDE } M_0} = \mathbb{C}1$

THEN $N' \cap M_1 = C^*(\hat{G})$

$N' \cap M_2 = \overline{C(G)\, C^*(G)} = B(\ell^2(G))$

$M_0' \cap M_2 = C^*(G)$

$\longrightarrow$ GOT $C^*(\hat{G})$ AND $C^*(G)$ OUT OF THE SEQUENCE.