CHRIS 6

BIG PICTURE

SUBFACTORS ← - - - - QUANTUM GROUPS

DRINFELD CENTRE

CHRIS

RYSZARD

MODULAR TENSOR CATEGORIES

TENSOR CATEGORIES $\xrightarrow{Z}$ MODULAR TENSOR CATEGORIES

TV

RT ↓

ANDRÉ

3d-TFT

TURAEV-VIRO (VIA TRIANGULATION)

RESHITIKIN-TURAEV

SLIGHTLY SMALLER PICTURE:

CATEGORY THEORY

FUSION

RIBBON $\xrightarrow{\text{COND}}$ MODULAR

TENSOR WITH DUALS $\xrightarrow{\text{COND}}$ RIBBON

TENSOR WITH DUALS $\xrightarrow{\text{COND}}$ FUSION

RIBBON $\xleftarrow{\text{COND}}$ BALANCED

BRAIDED $\xrightarrow{+\theta}$ BALANCED

TENSOR $\xrightarrow{+*}$ TENSOR WITH DUALS

TENSOR $\xrightarrow{+\beta}$ BRAIDED

CAT $\xrightarrow{+\otimes}$ TENSOR

Semi-simple with finitely many simples

ALG

RIBBON HOPF ALG

BALANCED V

HOPF ALGS S

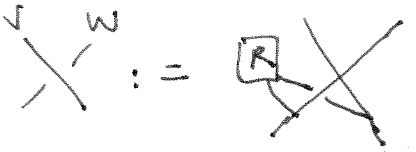
BRAIDED BIALG R

BIALGEBRA Δ

$U_q \mathfrak{g}$ AS ALG

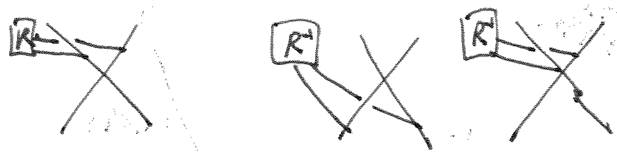
YESTERDAY ...

WARNING: WANT TO DEFINE A BRAIDING IN A-Mod



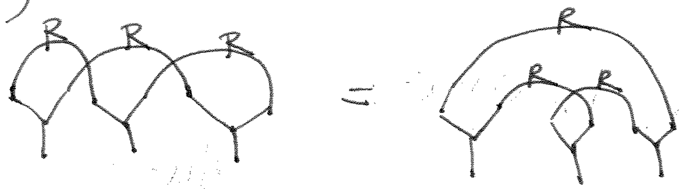
$m \otimes n \mapsto R(n \otimes m)$) WHAT WE USED

OTHER CONVENTIONS:



{ DIFFERENT AUTHORS USE DIFFERENT CONVENTIONS FOR "R" AND EVEN IN YESTERDAY'S LECTURE, DIFFERENT CONVENTIONS WERE USED ... WILL ALLOW ALL CONVENTIONS AT ONCE!

EXERCISE: 1) THE QUANTUM YANG-BAXTER EQUATION



CONVINCE YOURSELF THAT IF (A, R) SATISFIES Y-B, THEN $(A\text{-Mod}, \beta)$ SATISFIES REIDEMEISTER III.

2) OBSERVE THAT IN ANY BRAIDED TENSOR CAT, REIDEMEISTER III IS A CONSEQUENCE OF HEXAGON AXIOMS.

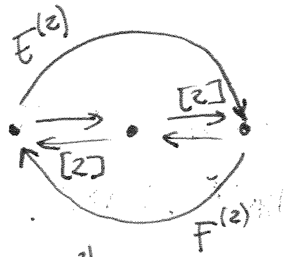
RECALL: $U_q^{res}\mathfrak{g} = \mathbb{Z}[q, q^{-1}]$-SUBALG OF $U_q\mathfrak{g}/\mathbb{C}(q)$

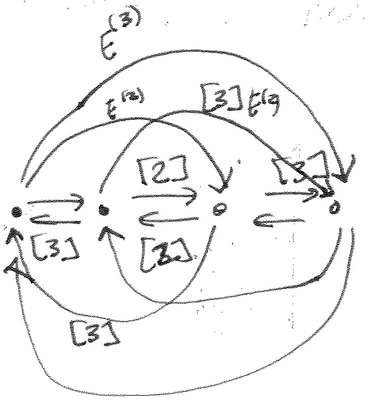
GENERATED BY $\left\{ \frac{E_i^k}{[k]!}, \frac{F_i^k}{[k]!}, K_i^{\pm 1} \right\}$

"$E^{(k)}$" "$F^{(k)}$"

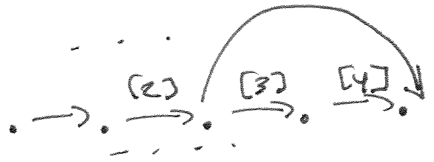
REPRESENTATIONS FOR RESTRICTED QUANTUM GPS

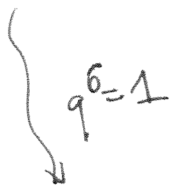
WEYL MODULES:

integral  

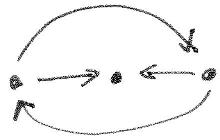
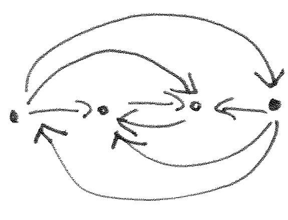


$$\text{"}\binom{[4]}{[2]}\text{"} = [3](q^2+q^{-2}) \leftarrow \frac{[3][4]}{[2]}E^{(2)}$$





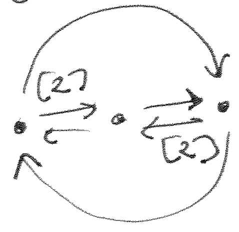
$q^6=1$

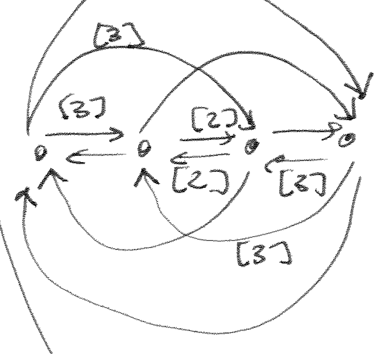
SPECIALIZE $(q^{2l}=1)$ e.g. $\boxed{q^4=1}$  

$[l]=0$

DUAL WEYL

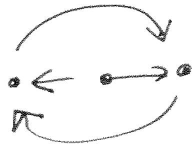
BY A NON-TRIVIAL COMPUTATION

INTEGRAL:  



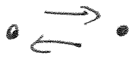
$q^4=1$

SPECIALIZE:



NOT ISOMORPHIC!

TILTING MODULES (=WITH WEYL AND DUAL WEYL FILTRATION)

$q^4=1$: 

EXERCISE: CHECK $W_3^* = W_3$ AS U^{res}-Mod AT $q^4=1$.

WANT A-Mod TO BE A RIBBON CATEGORY.

STRUCTURE ON A TO GET A DEHN TWIST ON A-Mod ?