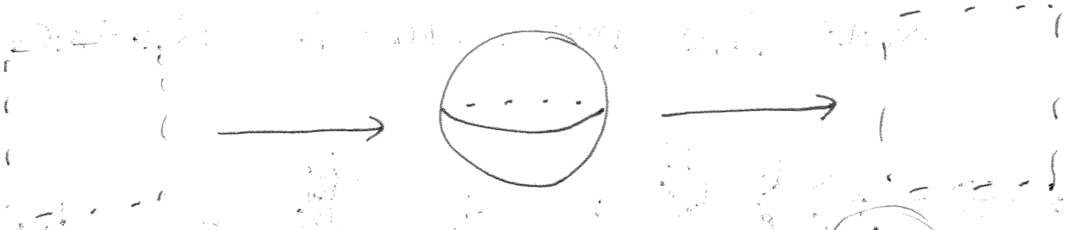
ANDRÉ 8 COMPUTING THE INVARIANTS

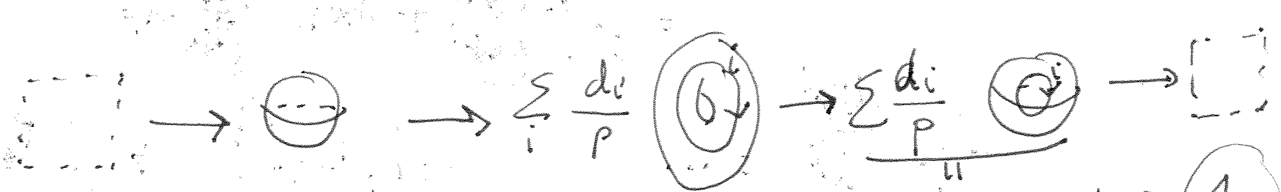
S^3:



$$1 \longmapsto 1 \xmapsto{\cdot \frac{1}{p}} \frac{1}{p}$$

$S^1 \times S^2$:

$d_i = \dim(C_i)$



$$1 \longmapsto 1 \longmapsto \sum_i \frac{d_i}{p} \longrightarrow \sum \frac{d_i}{p} = \sum \frac{d_i^2}{p} = p \longmapsto 1$$

SOLID TORUS ("OUTSIDE TORUS")

SOLID TORUS ("INSIDE TORUS")

$\Rightarrow$ FORM $S^1 \times S^2$ TOGETHER.

<u>REM</u>: $\left(\int_{S^2} Z\right)(M) := Z(M \times S^2)$ is A 1d-TQFT

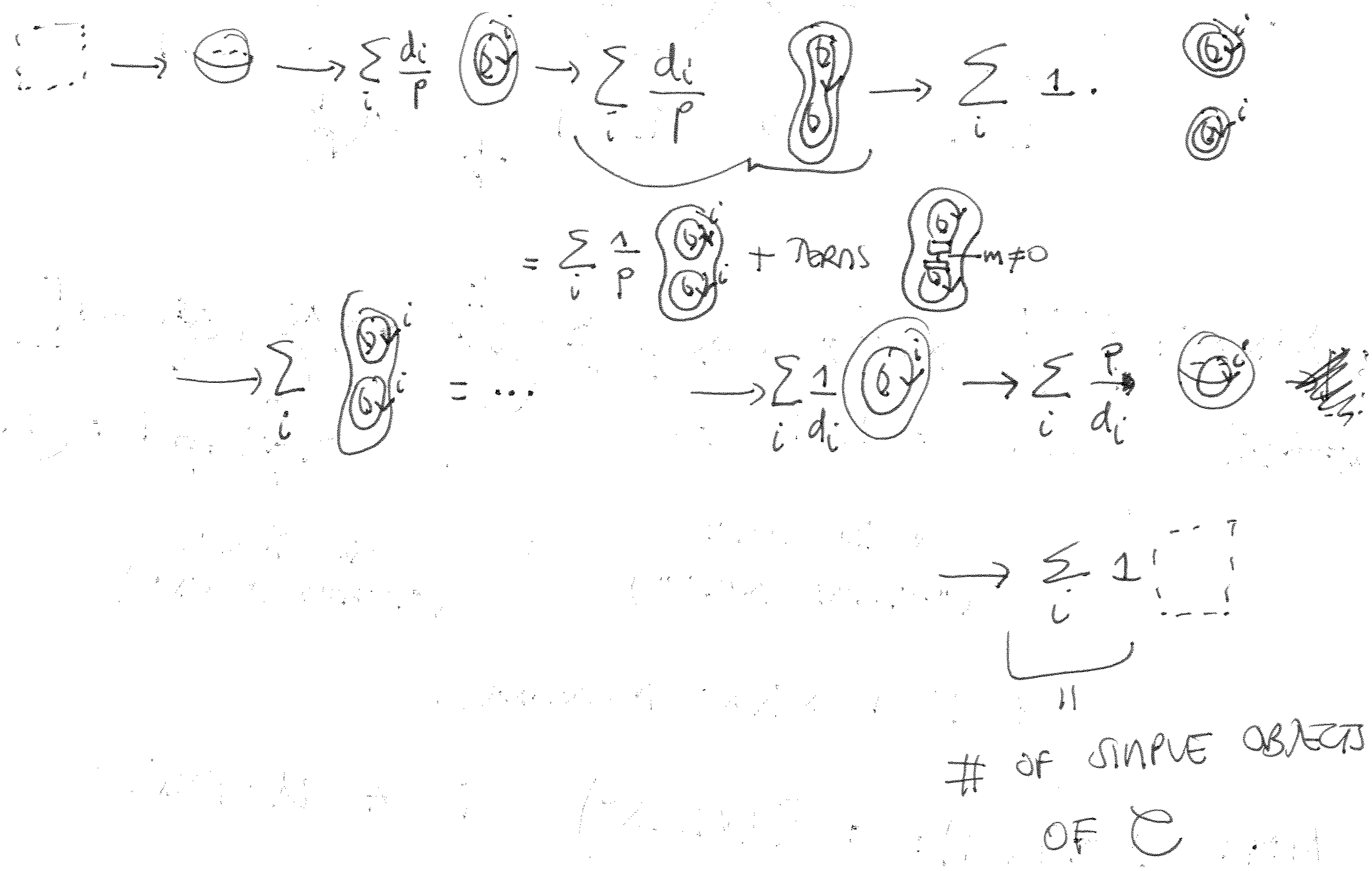
$$\left(\int_{S^2} Z\right)(pt) = Z(S^2) = \mathbb{C}$$

$$\left(\int_{S^2} Z\right)(S^1) = Z(S^1 \times S^2) \underset{\uparrow}{=} \dim\left(\int_{S^2} Z(pt)\right) = 1$$

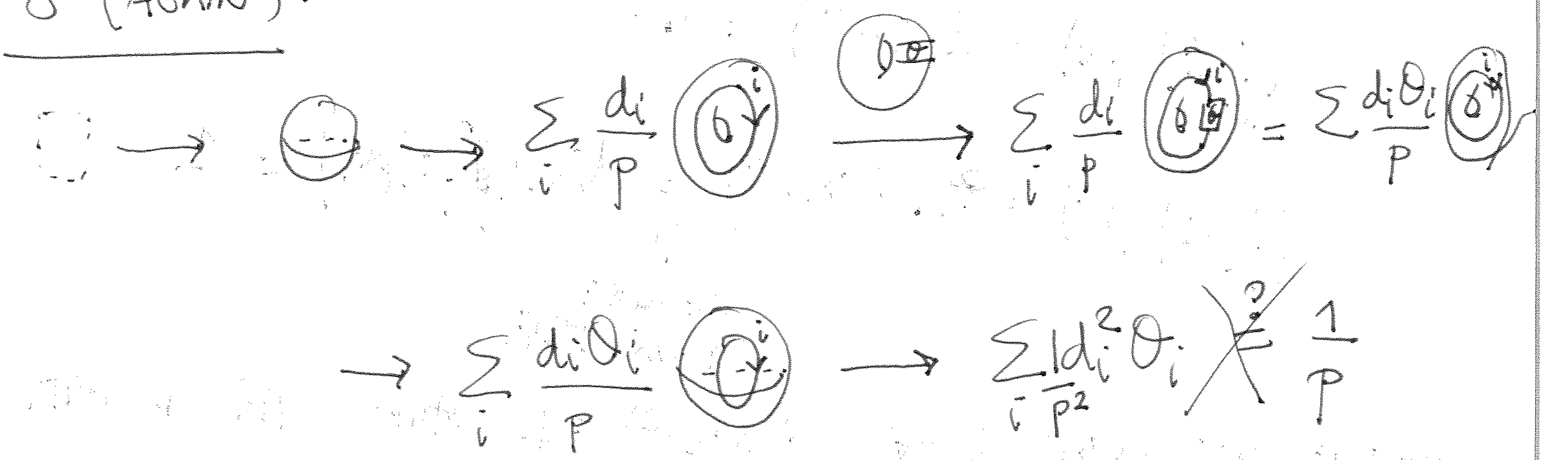
BECAUSE 1-dim TQFT

HENCE WE MUST HAVE $Z(S^1 \times S^2) = 1$ AND THIS IS WHY WE HAD TO HAVE $\sum \frac{d_i^2}{p} = p$

T^3: (MOVIE GIVING T^3: TORUS T^2 SUBMERGED IN THE WATER, COMES OUT AND BACK IN — WE SEE THE TORUS AND ITS REFLECTION)



S^3 (AGAIN):



→ THIS DIFFERENT COMPUTATION GAVE A DIFFERENT ANSWER THAN THE FIRST!

REASON: THESE 3-MFD INVARIANTS ARE IN FACT INVARIANTS OF 3-MFD | COBORDANT TO A 4-MFD!
| i.e. EQUIPPED WITH A BOUNDING 4-MFD

THM (BARTLETT, DOUGLAS, SCHOMMER-PRIES, VICARY)

THERE IS A 1-1 CORRESPONDENCE BETWEEN SYM. MON. FUNCTORS $\mathrm{Bord}^{\partial}_{1,2,3} \longrightarrow \mathrm{LinCat}$ s.t. $1 \in$ ~~[illegible]~~ $\mathcal{C} = Z(S^1)$

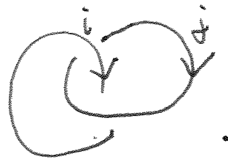
(i.e. $\mathrm{Bord}_{1,2,3}$ + EVERYTHING ~~EQUIPPED~~ WITH ~~COBORDANT~~ BOUNDING 4-MFD ($\frac{3}{2}$) UP TO COBORDISMS (FOR THE 4-MFDS))

EXTRA DATA = "ANOMALI" $\longleftrightarrow$

IS SIMPLE, AND PAIRS $(\mathcal{C}, p)$ WHERE $\mathcal{C}$ IS A MODULAR TENSOR CATEGORY AND p IS A SQUARE ROOT OF $\sum d_i^2$ FOR $d_i = \dim(C_i)$

SIMPLE OBJ. OF $\mathcal{C}$.

Def. A RIBBON CATEGORY IS MODULAR IF THE MATRIX

$\Big[$  $\Big]_{ij}$ HAS NON-ZERO DETERMINANT.

(COMES FROM $\partial \xrightarrow{\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}} \partial$ CAN WRITE IT AS A SEQUENCE OF (NON-INVERTIBLE) SIMPLE MOVES.

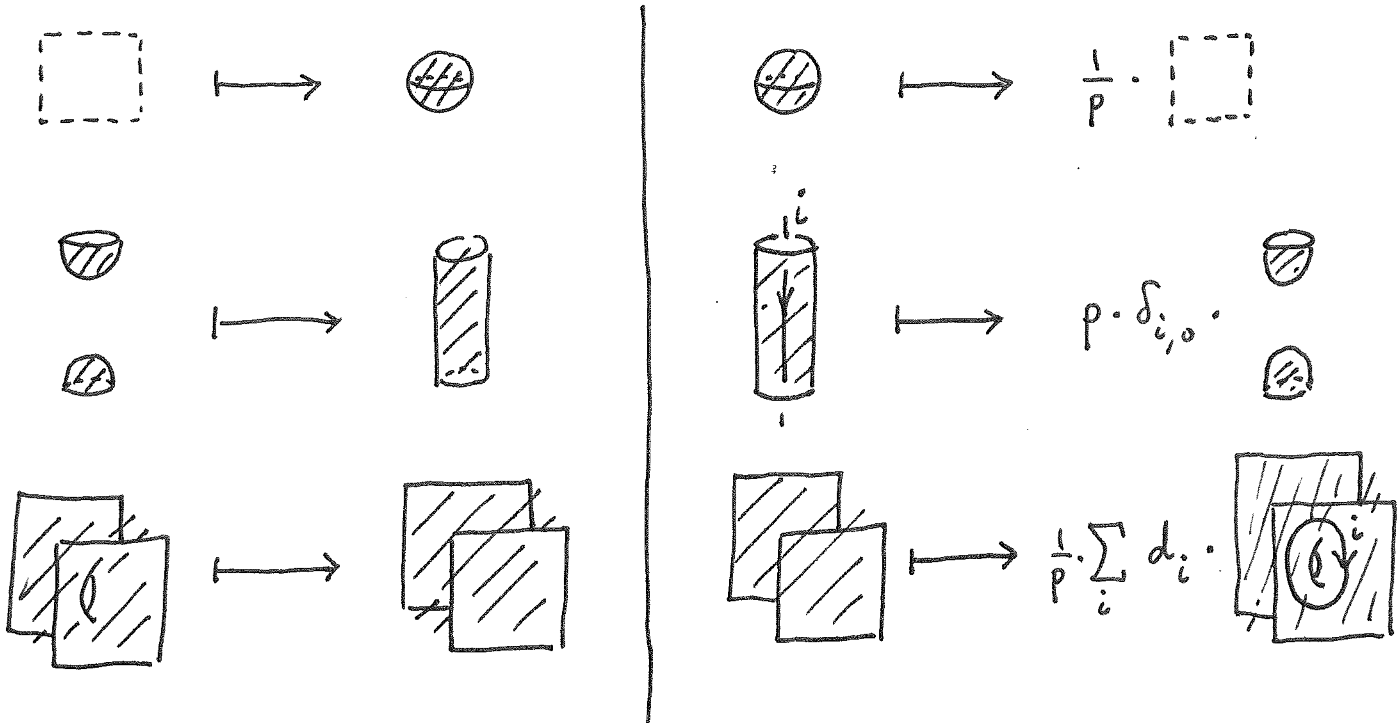
$\rightsquigarrow$ GET A MATRIX, WHICH HAS TO BE INVERTIBLE AS THE COMPOSED MAP IS INVERTIBLE.

EXERCISE **: $p_+ = \sum_i d_i^2 \theta_i = p \cdot e^{2\pi i \frac{3k}{8(k+2)}}$

PROVE THAT!

IF $p = p_+$, THE ANOMALI IS TRIVIAL $\Leftrightarrow$ $\mathrm{Bord}_{1,2,3} \to \mathrm{LinCat}$ FUNCTOR.

The local moves



where $d_i = \bigcirc^{i}$ and $p^2 = \sum_i d_i^2$

$\mathrm{Rep}^{ss}(U_q\,\mathfrak{sl}(2))$

	$q^6=1$	$q^8=1$	$q^{10}=1$
d_i	1 1	1 $\sqrt{2}$ 1	1 ϕ ϕ 1
$p=\sqrt{\sum_i d_i^2}$	$\sqrt{2}$	2	$\sqrt{\sqrt{5}+5}$
$\frac{1}{2\pi}\cdot\arg(\theta_i)$	0 $1/4$	0 $3/16$ $1/2$	0 $3/20$ $2/5$ $3/4$
$p_+ = \sum_i d_i^2\theta_i$	$1+i$	$2\cdot e^{2\pi i\cdot\frac{3}{16}}$	$(\sqrt{5}+5)\cdot e^{2\pi i\cdot\frac{9}{40}}$

"$\mathfrak{sL}(2)$ level k": $q^{2(k+2)}=1$; $d_i=[i+1]_q$; $\theta_i = q^{\frac{1}{2}i^2+i}$; $p_+ = p\cdot e^{2\pi i\frac{3k}{8(k+2)}}$