ANDRÉ 7

RECALL:

$F \dashv G$ ADJOINT FUNCTORS $\quad F : \mathcal{C} \rightleftarrows \mathcal{D} : G$

$\{c_i\} \quad \{d_j\}$ BASES

$F \rightsquigarrow (A_{ij})$

$G \rightsquigarrow (A^*_{ij})_{ji}$

COUNIT OF THE ADJUNCTION ON SIMPLE OBJECTS:

$$FG(d_j) \longrightarrow d_j$$

$$\| $$

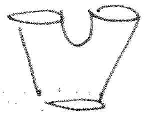
$$F\Big(\bigoplus_k A^*_{kj} \otimes c_k\Big) = \bigoplus_{k,l} A^*_{kj} \otimes A_{kl} \otimes d_l \longrightarrow d_j$$

$$\text{is } \begin{cases} \bigoplus_k A^*_{kj} \otimes A_{kj} \xrightarrow{\sum_k ev} \mathbb{C} & (l = j) \\ 0 & l \neq j \end{cases}$$

UNIT OF THE ADJUNCTION:

$$c_i \longrightarrow GF(c_i) = \bigoplus_{j,l} A_{ij} \otimes A^*_{lj} \otimes c_l$$

$$\text{is } \begin{cases} \mathbb{C} \xrightarrow{\sum_j coev} \bigoplus_j A_{ij} \otimes A^*_{ij} & i = l \\ 0 & i \neq l \end{cases}$$

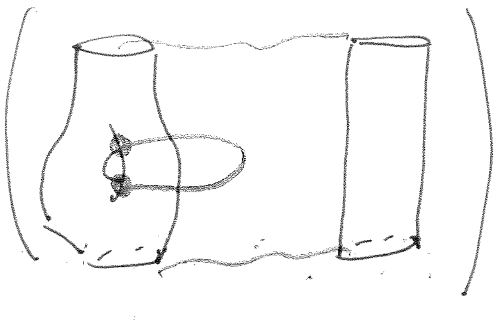
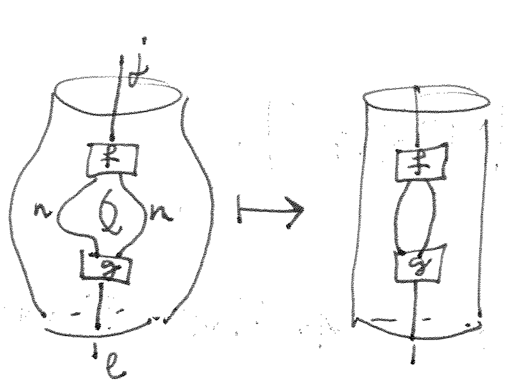
RECALL: $Z($  $) \dashv Z($  $)$

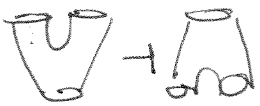
COPROD Δ $\qquad$ PROD m

AND $Z($  $) \dashv Z($  $)$

COUNIT $\qquad$ UNIT

CLAIM:

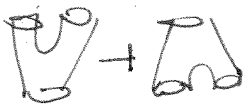
Z () is 

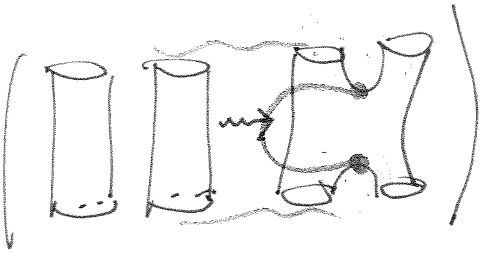
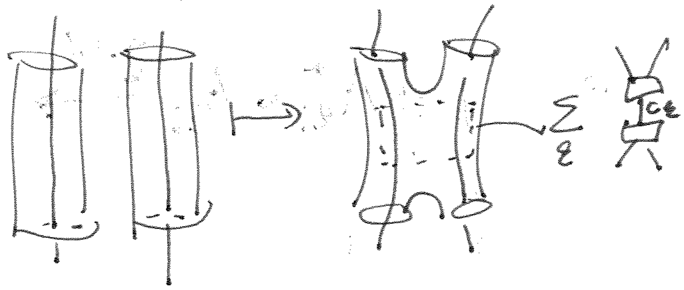
↑ COUNIT OF 

EXPLICITLY:
~~[illegible]~~ ($\ell = j$ OTHERWISE NO MAP)

CLAIM: $\bigoplus_{n,m} \mathrm{Hom}(c_j, c_n \otimes c_m) \otimes \underbrace{\mathrm{Hom}(c_j, c_n \otimes c_m)^*}_{= \, \mathrm{Hom}(c_n \otimes c_m, c_j)} \longrightarrow \mathbb{C} = \mathrm{Hom}(c_j, c_j)$

COMPOSITION IN $\mathcal{C}$

UNIT OF  :

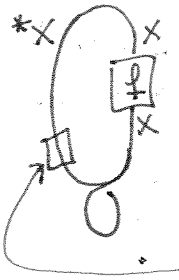
Z () is 

COEV $\in \bigoplus_{k} \mathrm{Hom}(c_i \otimes c_j, c_k) \otimes \mathrm{Hom}(c_k, c_i \otimes c_j)$

COMPOSES TO $\mathrm{id}_{c_i \otimes c_j}$

FOR  , NOTHING HAPPENS ON NOTHING...

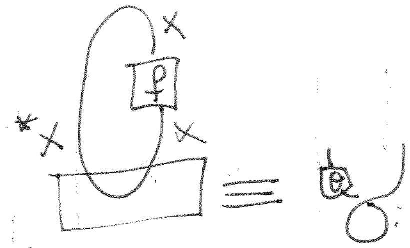
QUANTUM TRACE:

$f : X \to X$  GIVES A TRACE BUT $tr(f \otimes g) \neq tr(f) \cdot tr(g)$

FIX BY ADDING A θ (OR θ^{-1}?)

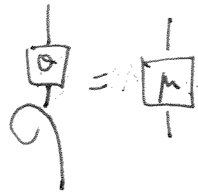
EXERCISE: FIGURE OUT WHICH WORKS TO GIVE =.

CONVENTION:



AND ALWAYS ONLY USE LEFT DUALS

(NOTE:



IN CHRIS' LECTURE)

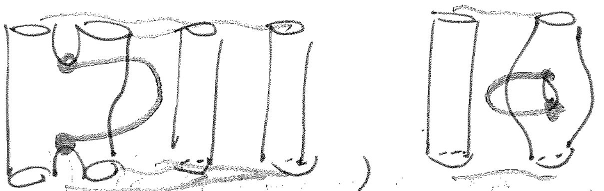
$\dim(X) =$

$= tr(id_X)$

EXERCISE: $tr(fg) = tr(gf)$

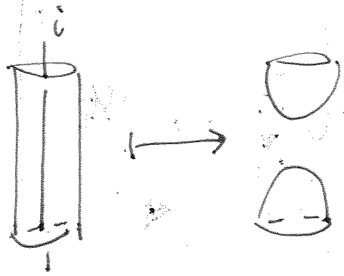
NEXT GOAL: COMPUTE Z ON

① , ②



③ , ④

①



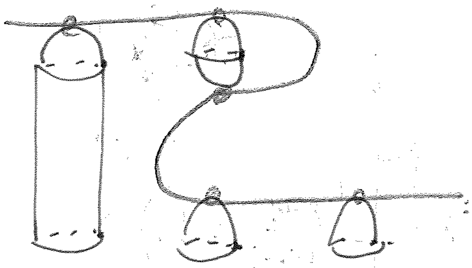
$p \cdot \delta_{i,0}$

{ ONLY NON-ZERO FOR i INDEXING THE UNIT OBJECT.

p IS DETERMINED BY Z BUT NOT FROM THE BALANCED CATEGORY ITSELF.

CONDITION ON p: $p^2 = \sum_i \dim(c_i)^2$

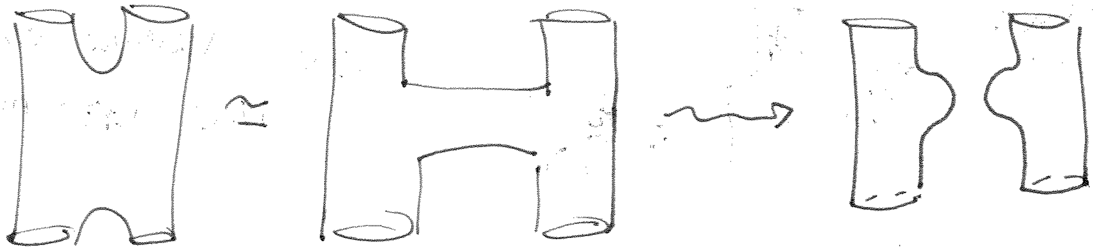
②



$1 \longmapsto p \longmapsto 1$

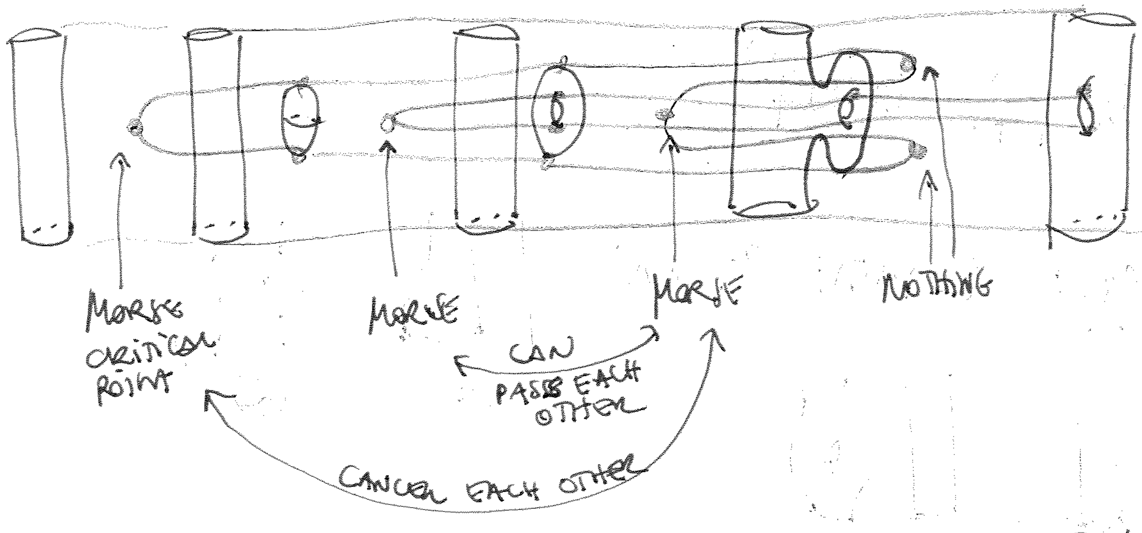
$\cdot \frac{1}{p}$

(3)

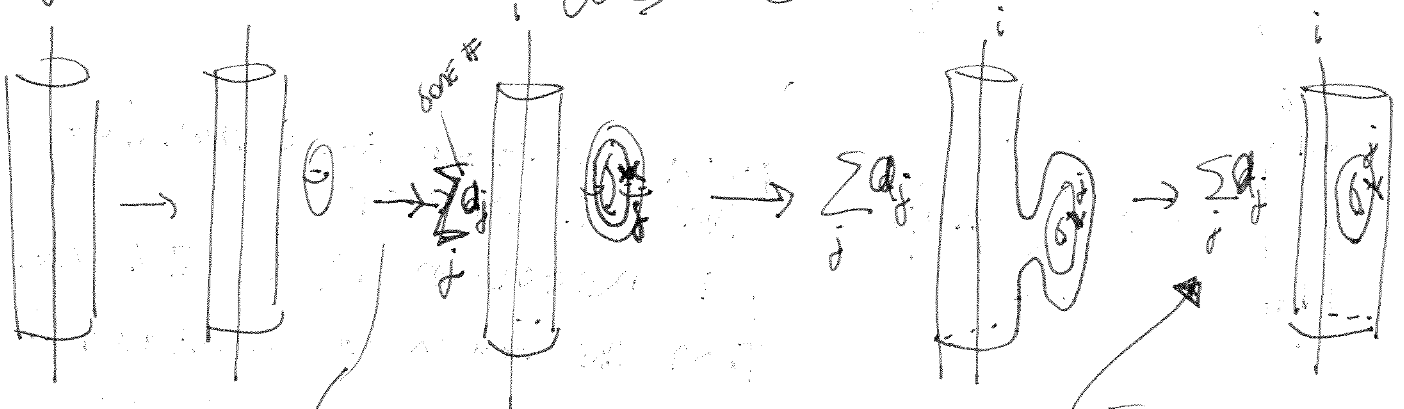


SAME AS (1)

(4) WRITE (4) AS A COMPOSITE:

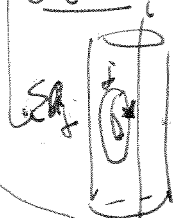


$\rightsquigarrow$ SAME AS THE 3-MFD IN (4)

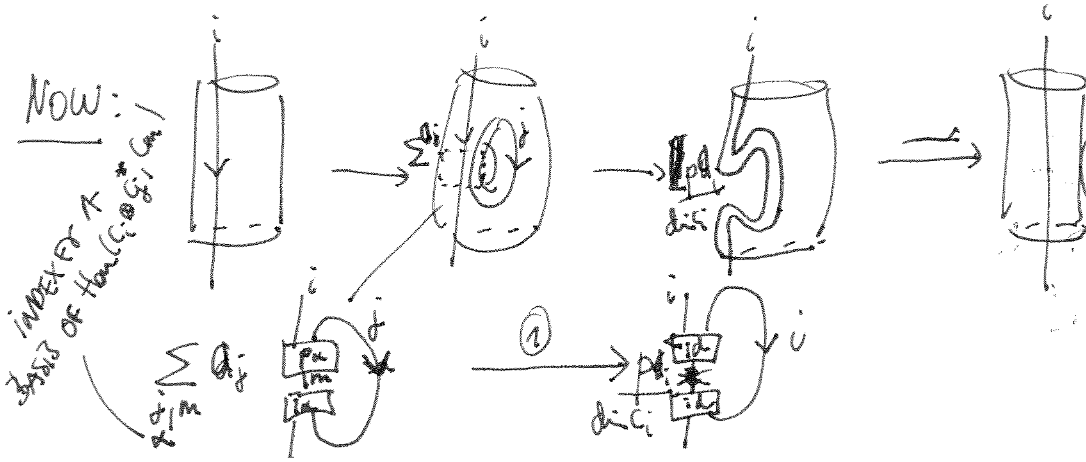


$$\mathrm{Hom}(1, C_j \otimes C_\ell) = \begin{cases} 0 & \text{NOT DUAL} \\ \mathbb{C} & \text{DUAL} \end{cases}$$

CLAIM: EQUAL TO



NOW:



$$\frac{p\,a_i}{d_i c_i} = 1$$

$$\Rightarrow \boxed{a_i = \frac{d_i c_i}{p}}$$