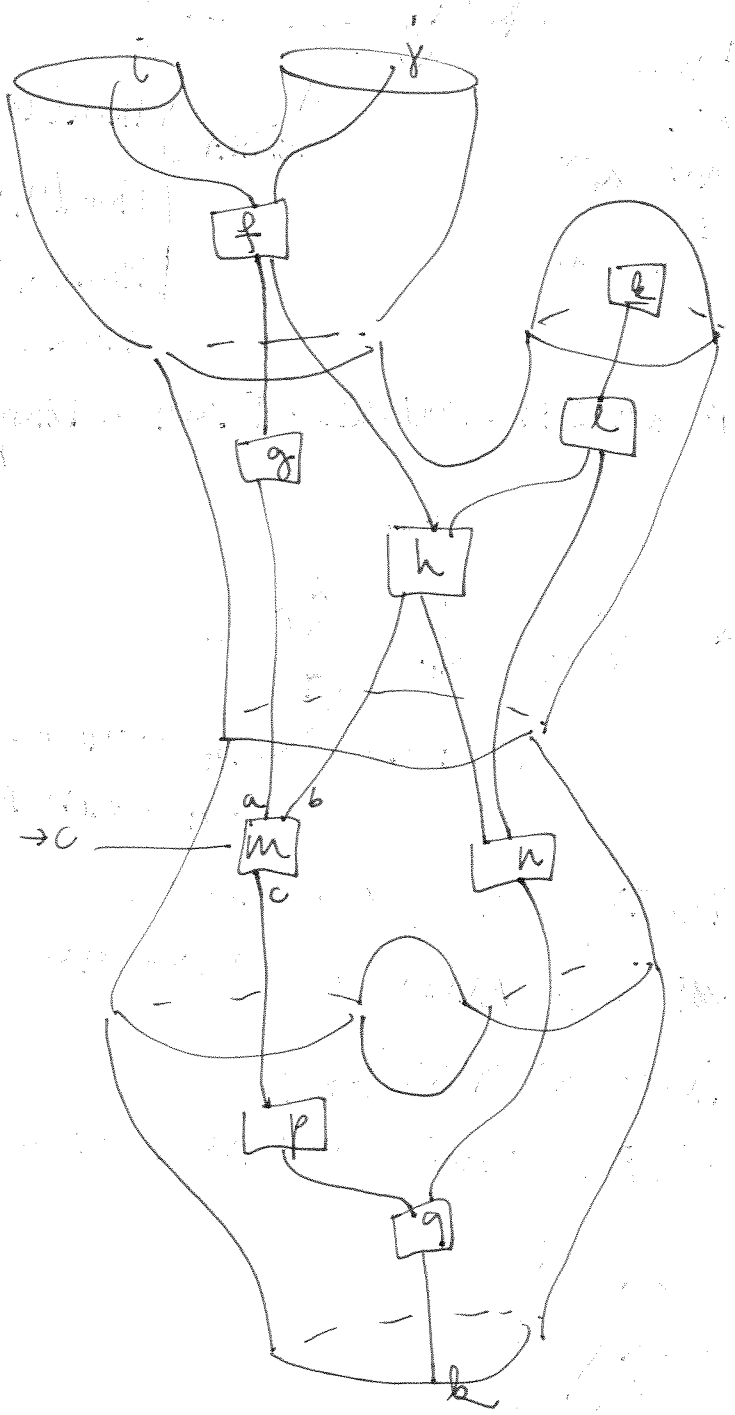
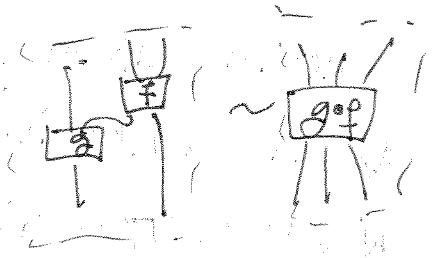
ANDRE 6

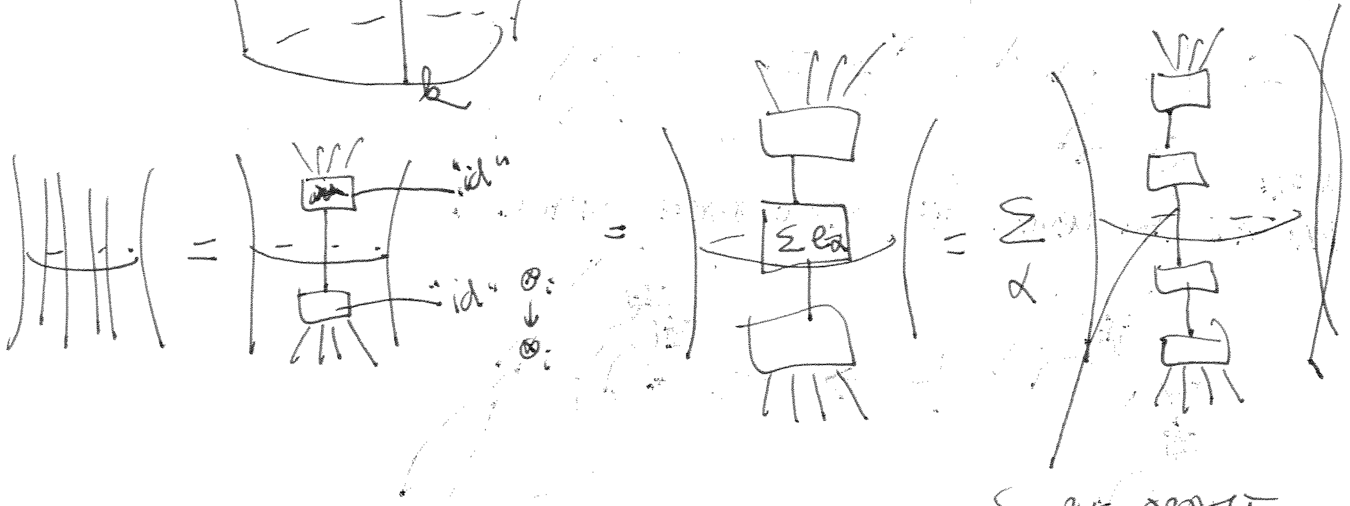
$m : a \otimes b \to c$



Claim: The structure space of the composite functor $\mathcal{C} \to \mathcal{C} \boxtimes \mathcal{C}$ can be described as the linear span of the set of "internal string diagrams" modulo isotopy and the relation of local equivalence:



IDEA:



SIMPLE OBJECT

Space of internal diagrams:

$\oplus$ Labelings of the circles by basis elements of $\mathcal{C}$

$\otimes$ All pants, copants, cups and caps in the decomposition

$\mathrm{Hom}_{\mathcal{C}}(\ldots, \ldots)$

$\uparrow$ either $\left\{ \begin{array}{l} \mathrm{Hom}(a\otimes b, c) \\ \mathrm{Hom}(a, b\otimes c) \\ \mathrm{Hom}(a, 1) \\ \mathrm{Hom}(1, a) \end{array} \right.$

[Will appear in a paper of Bartels-Douglas-Vicary-Schommer-Pries]

$$\begin{tikzcd} \emptyset \arrow[r, bend left, "\emptyset"] \arrow[r, bend right, "\emptyset"'] \arrow[r, phantom, "M^3"] & \emptyset \end{tikzcd} \xrightarrow{\ Z\ } \begin{tikzcd} \mathrm{Vect} \arrow[r, bend left, "\otimes\mathbb{C}"] \arrow[r, bend right, "\otimes\mathbb{C}"'] \arrow[r, phantom, "\Downarrow"] & \mathrm{Vect} \end{tikzcd}$$

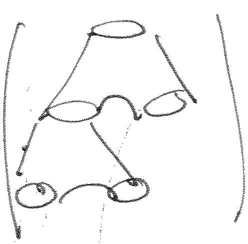
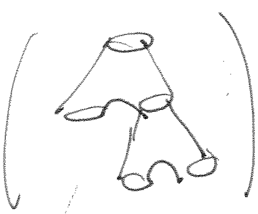
A number!

$\longrightarrow$ A 3-mfd invariant

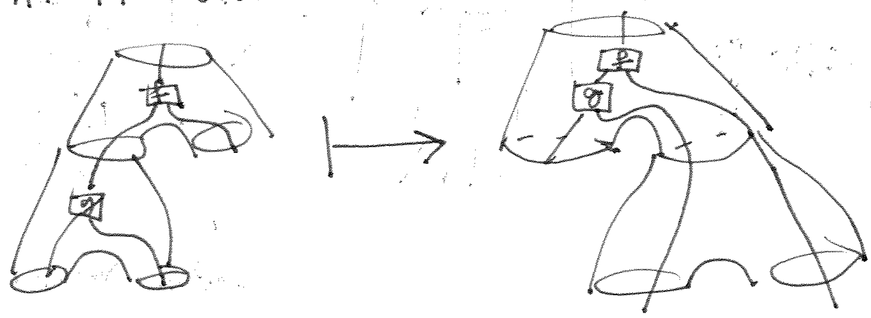
To compute these invariants, will decompose M^3 as a composition $\rightsquigarrow$ need to understand the natural transformations of functors

$\equiv$ linear maps between structure spaces.

Ex:

$Z \left(\right.$  $\left. \right) \overset{\simeq}{\Rightarrow} Z \left(\right.$  $\left. \right)$

Claim:
Map at the level of structure spaces:



PROOF: RECALL THAT $v \in A^i_{jkl}$ IS A MAP $C_i \to F(C_j, C_k, C_l)$

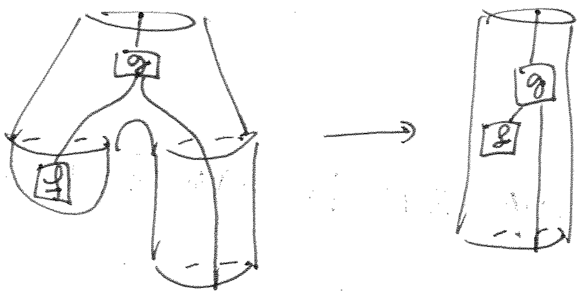
LEFT HAND SIDE: $C_i \xrightarrow{f} C_m \otimes C_l \xrightarrow{g \otimes 1} (C_j \otimes C_k) \otimes C_l$ $= F(C_j, C_k, C_l)$

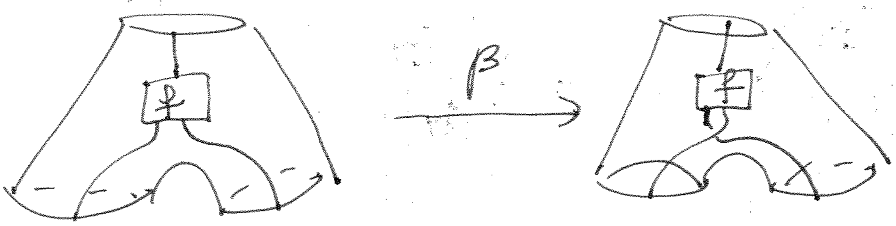
POST-COMPOSE WITH ASSOCIATOR

RHS: $C_i \xrightarrow{g \circ f} C_j \otimes (C_k \otimes C_l)$ $= G(C_j, C_k, C_l)$

AND THIS IS WHAT THE NATURAL TRANSFORMATION DOES.

EX:



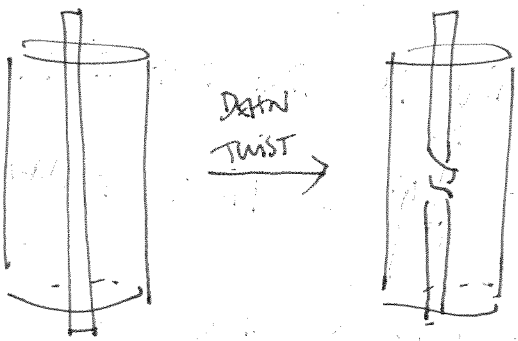


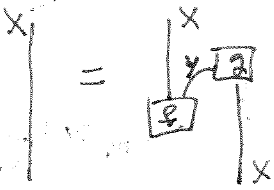
PROOF:

LHS: $f : C_i \to C_j \otimes C_k$

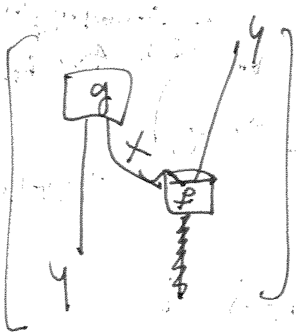
RHS: $C_i \xrightarrow{\beta \circ f} C_k \otimes C_j$

EX



LEMMA: IF IN SOME IDEMPOTENT COMPLETE CATEGORY $\mathcal{C}$, ONE CAN WRITE 1_X AS 

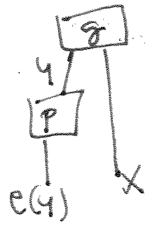
, THEN X IS DUALIZABLE.

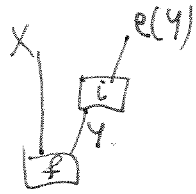
PF: $e :=$ 

SATISFIES $e^2 = e$

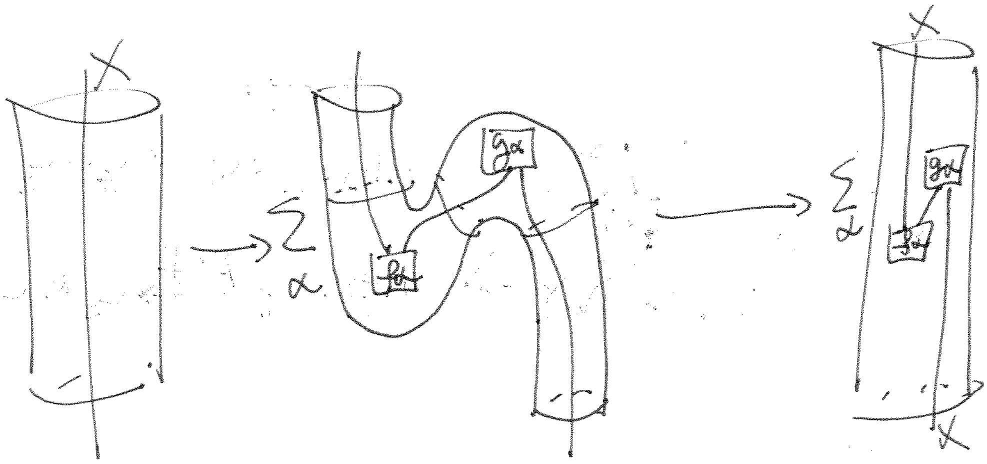
$\Rightarrow \exists\, e(Y)$.

THEN $e(Y)$ IS A DUAL OF X

AND $ev =$ 

$coev =$ 

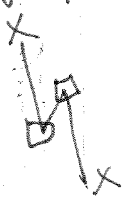
WANT TO SHOW THAT EVERY OBJECT IS DUALIZABLE IN $\mathcal{C}$.



(MAY ASSUME X SIMPLE)

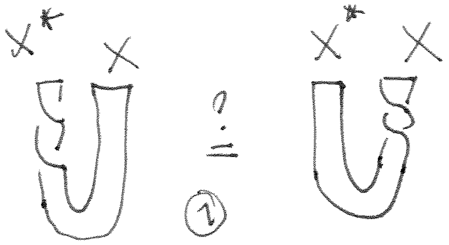
THE COMPOSITE NATURAL TRANSFORMATION IS THE IDENTITY.

$\Rightarrow$ WRITE 

AS A SUM OF 

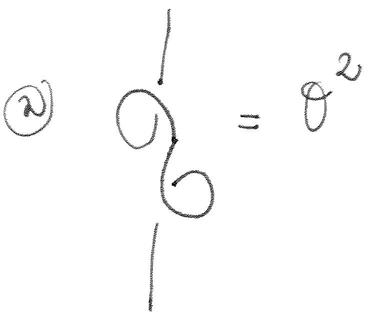
. PICK A NON-ZERO TERM AND SCALE IT TO GET A MAP $X \to X$

$\to$ DONE BY THE LEMMA.



if ① and ② ~~these two are the same~~, we call this a RIBBON CATEGORY

$=$ COMPATIBLE $\underbrace{\text{BALANCED}}_{=\ \text{BRAIDED + TWISTS}}$ + $\underbrace{\text{RIGID}}_{=\ \text{HAS DUALS.}}$



COME FROM