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ANDRE 5

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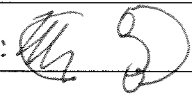
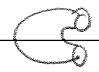
$\mathcal{C} = Z(S^1)$ FOR SOME $Z : \mathrm{Bord}_{1,2,3} \longrightarrow \mathrm{LinCat}$

Ev: 

CoEv: 

$\mathcal{C} \longrightarrow \mathrm{Fun}(\mathcal{C}, \mathrm{Vect})$

$v \longmapsto Ev(v, -)$

$\mathrm{Fun}(\mathcal{C}, \mathrm{Vect}) \longrightarrow \mathcal{C}$

$F \longmapsto F \otimes 1(\mathrm{coEv})$

ARE INVERSE EQUIV.

$\Rightarrow \mathcal{C}$ IS AN ABELIAN CATEGORY.

$X \in \mathcal{C}$, $A := \mathrm{End}_{\mathcal{C}}(X)$. IF $A \neq \mathbb{C}$, $\exists a \in A$, $a \neq 0$ NOT INVERTIBLE.

$a : X \to X$ ⇝ SES ⇝ SPLITS ⇝ X SPLITS AS A DIRECT SUM ...

$\Rightarrow \boxed{\mathcal{C} \simeq \mathrm{Vect}^{\oplus n}}$

A BASIS OF $\mathcal{C}$ IS A COLLECTION OF REPRESENTATIVES OF THE SIMPLE OBJECTS.

$F : \mathcal{C} \longrightarrow \mathcal{D}$ FUNCTOR GIVEN BY $F(c_i) = \bigoplus_j A_{ij} d_j$

$\{c_i\}$ $\{d_j\}$ BASES

FOR SOME MATRIX OF VECTOR SPACES.

$\left(A_{ij} = \mathrm{Hom}_{\mathcal{D}}(d_j, F(c_i)) \right)$.

$F \dashv G$ (F ADJOINT TO G)

(A_{ij}) THEN $G \leftrightarrow (A_{ij}^*)_{ji}$ " CONJUGATE TRANSPOSE " IN POSITION ji.

DUAL VECTOR SPACE

NEED $\mathrm{Hom}(F(X), Y) \overset{?}{\cong} \mathrm{Hom}(X, G(Y))$

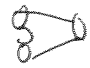
ENOUGH FOR BASIS ELEMENTS.

$$\mathrm{Hom}(F(c_i), d_j) = \mathrm{Hom}\Big(\bigoplus_k A_{ik} \otimes d_k, d_j\Big) = \mathrm{Hom}(A_{ij} \otimes d_j, d_j)$$
$$= \mathrm{Hom}(A_{ij}, \mathbb{C}) = A_{ij}^*$$

$$\mathrm{Hom}(c_i, G(d_j)) = \mathrm{Hom}\Big(c_i, \bigoplus_k A_{kj}^* \otimes c_k\Big) = \mathrm{Hom}(c_i, A_{ij}^* \otimes c_i)$$
$$= \mathrm{Hom}(\mathbb{C}, A_{ij}^*) = A_{ij}^* \quad \checkmark$$

CALL THE A_{ij}'S THE "STRUCTURE SPACES" OF THE FUNCTOR

CONSIDER NOW $m: \mathcal{C} \otimes \mathcal{C} \longrightarrow \mathcal{C}$. HAS STRUCTURE



SPACES

$$A_{ij}^k = \mathrm{Hom}_{\mathcal{C}}(c_k, c_i \otimes c_j).$$

PRODUCT IN $\mathcal{C}$

$$\Delta: \mathcal{C} \longrightarrow \mathcal{C} \otimes \mathcal{C}$$

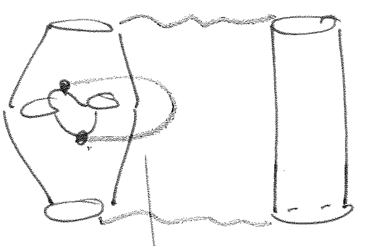
WILL SEE THAT THERE IS AN "ADJUNCTION"

 $\dashv$ 

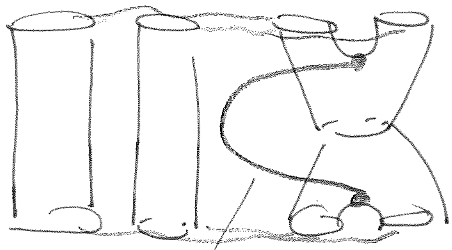
m Δ

UNIT AND COUNIT OF THE ADJUNCTION:

COUNIT



UNIT



COBORDISM CANCELLING THE TWO CRITICAL POINTS.

COBORDISM CREATING THESE TWO CRIT. POINTS.

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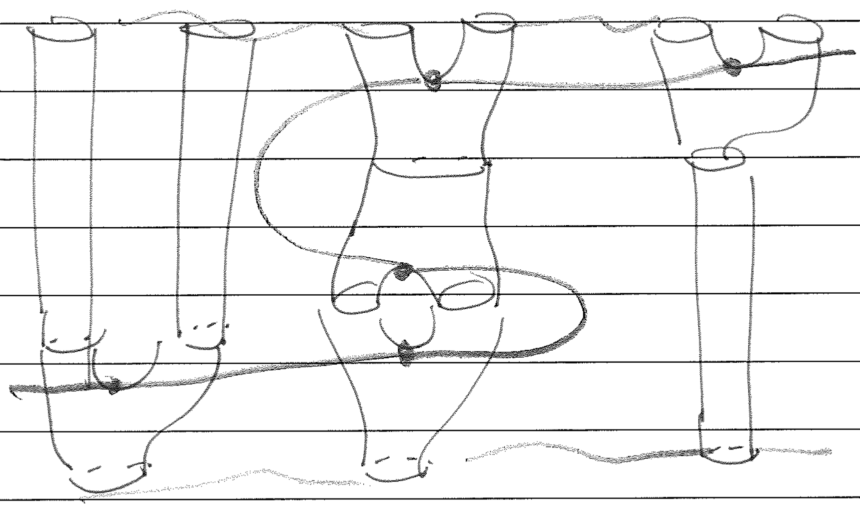
NEED TO SATISFY A COUPLE OF EQUATIONS:



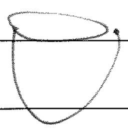
AND FLIP ...

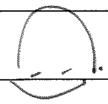
$\rightsquigarrow$ Δ IS THE ADJOINT (/ "DUAL") OF m.

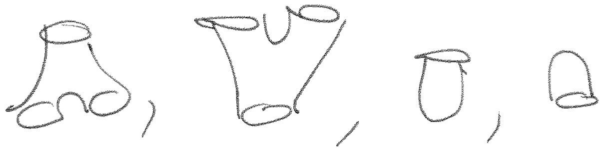
AND THE STRUCTURE SPACES FOR Δ ARE

$$A^{ij}_{k} = \mathrm{Hom}_{\mathcal{C}}(C_k, C_i \otimes C_j)^* = \mathrm{Hom}_{\mathcal{C}}(C_i \otimes C_j, C_k)$$

$$\Big(\underset{\substack{\uparrow \\ \text{DEF OF } A^{ij}_{k}}}{=} \mathrm{Hom}_{\mathcal{C}\boxtimes\mathcal{C}}(C_i \boxtimes C_j, \Delta(C_k)) \Big)$$

 $A^i = \mathrm{Hom}(C_i, 1)$ (EASY)

 $A_i = \mathrm{Hom}(1, C_i)$ (USING AN ADJUNCTION)

 GENERATE ALL SURFACES

$\rightsquigarrow$ GET A FORMULA FOR THE STRUCTURE CONSTANTS/SPACES OF ANY FUNCTOR ...

AND THEN DESCRIBE THE NATURAL TRANSFORMATIONS.