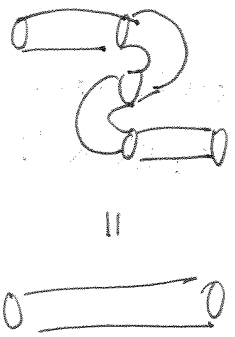
ANDRÉ 4

THM: $\mathcal{C} = Z(S^1) \Rightarrow \mathcal{C}$ IS SEMI-SIMPLE

PROOF:

WILL ONLY USE



$$V \longmapsto V \otimes \bigoplus_{i=1}^{n} X_i \otimes Y_i$$

$$\longmapsto \bigoplus_{i=1}^{n} \mathrm{EV}(V, X_i) \cdot Y_i \simeq V$$

RETRACT OF SUCH

VECTOR SPACE

ACTION ON Y_i

① HENCE, THE Y_i'S, $i = 1, \dots, n$, SPAN $\mathcal{C}$ UNDER $\oplus$ AND RETRACTS.

② THE HOM-SPACES ARE FINITE DIMENSIONAL:

$$\begin{array}{ccc} V & & \bigoplus_{i=1}^{n} \mathrm{EV}(V, X_i) \cdot Y_i \\ \downarrow f & \longmapsto & \downarrow \oplus \underbrace{\mathrm{EV}(f, X_i)} \otimes \mathrm{id}_{Y_i} \\ W & & \bigoplus_{i=1}^{n} \mathrm{EV}(W, X_i) \cdot Y_i \end{array}$$

LIVES IN THE FINITE DIM. VECT SPACE $\mathrm{Hom}(\mathrm{EV}(V, X_i), \mathrm{EV}(W, X_i))$

③ SHORT EXACT SEQUENCES ARE SPLIT:

$$\begin{array}{ccc} 0 & & 0 \\ \downarrow & & \downarrow \\ A & & \oplus\, \mathrm{EV}(A, X_i) \cdot Y_i \\ \downarrow & & \downarrow \oplus \varphi_i \otimes 1_{Y_i} \\ B & \longmapsto & \oplus\, \mathrm{EV}(B, X_i) \cdot Y_i \\ \downarrow & & \downarrow \oplus \psi_i \otimes 1_{Y_i} \\ C & & \oplus\, \mathrm{EV}(C, X_i) \cdot Y_i \\ \downarrow & & \downarrow \\ 0 & & 0 \end{array}$$

SHORT EXACT SEQUENCE $\otimes Y_i$ OF VECTOR SPACES AND THE SPLITTING IN Vect GIVES A SPLITTING AFTER $\otimes Y_i$.

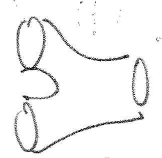
④ EACH OBJECT IS A ⊕ OF SIMPLE OBJECTS

= OBJECT WHOSE ENDOMORPHISM ALGEBRA IS $\mathbb{C}$.

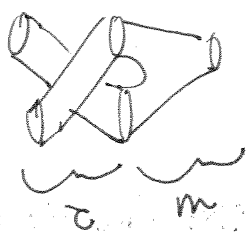
— BY ARTIN–WEDDERBURN USING ② & ③

HENCE WE ONLY HAVE TO CONSIDER THESE VERY SPECIAL TYPES OF LINEAR CATEGORY.

<u>FURTHER STRUCTURE</u>:



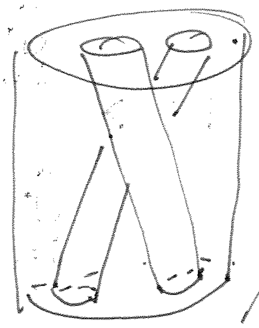
IS



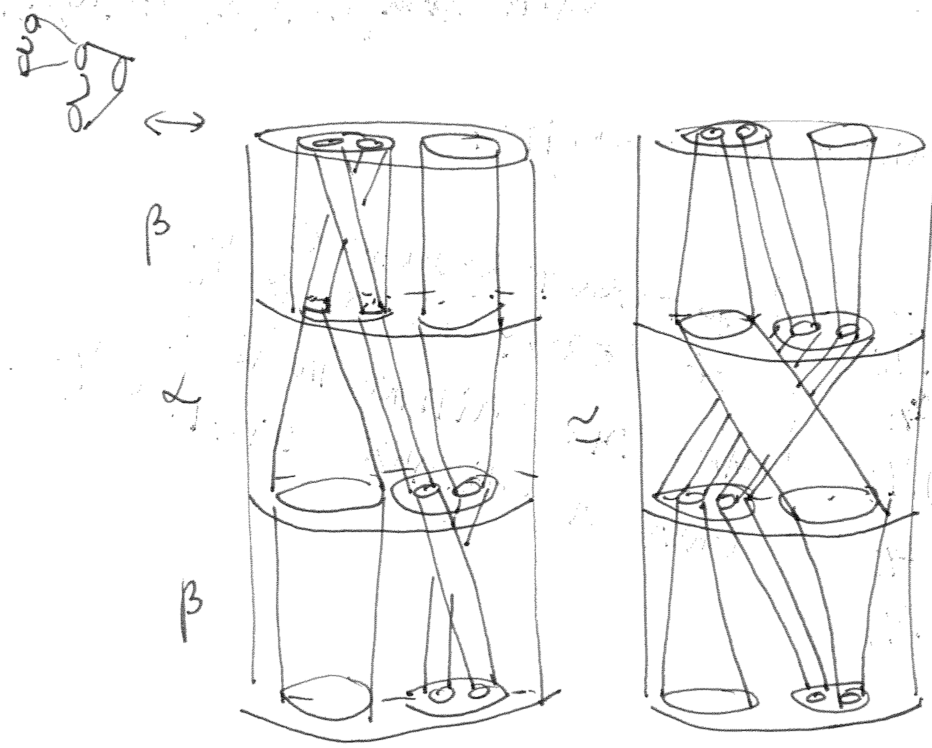
⇝ NATURAL TRANSFORMATION

$$\beta : m \circ \tau \Longrightarrow m$$

ACTUAL CHOSEN DIFFEO: COBORDISM



VERTICAL BOUNDARY



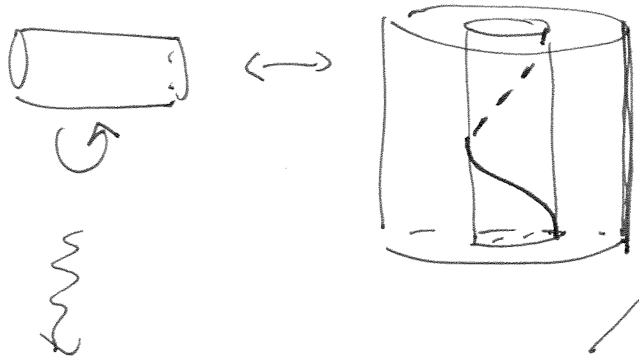
α ASSOCIATOR

β BRAIDING

α

$\Longrightarrow \mathcal{C}$ IS A BRAIDED ⊗-CATEGORY.

DEHN TWIST



VERTICAL BDRY "STRAIGHT ON OUTSIDE" AND TWISTED INSIDE"

NATURAL TRANSFORMATION

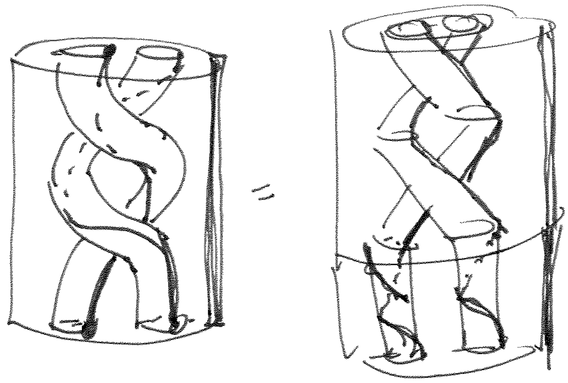
$$\mathbb{1}_{\mathcal{C}} \Longrightarrow \mathbb{1}_{\mathcal{C}}$$ — IDENTITY FUNCTOR ON $\mathcal{C}$

i.e. FOR EACH OBJECT X, AN AUTOMORPHISM $\theta_X : X \to X$

COMPATIBLE WITH THE DECOMPOSITION AS DIRECT SUMS

→ ENOUGH TO GIVE ON SIMPLE OBJECTS, FOR WHICH IT IS A NUMBER.

$$\theta_{X\otimes Y} = (\theta_X \otimes \theta_Y)\circ \beta^2$$



⇝ "BALANCED CATEGORY"

PICTURE FOR θ_X:

X

X

→ GRAPHICAL CALCULUS FOR BALANCED CATEGORIES: