# Homological Scaffolds of Brain Functional Networks

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## Abstract

We study the characteristics of functional brain networks at the mesoscopic level from a novel perspective that highlights the role of inhomogeneities in the fabric of functional connections. We do this by focusing on the persistent homology associated with the weighted functional network. We leverage this topological information to define the homological scaffolds, designed to summarise compactly the homological features of the correlation network and simultaneously make their homological properties amenable to networks theoretical methods. As a proof of principle, we apply these tools to compare resting-state functional brain activity in 15 healthy volunteers after intravenous infusion of placebo and psilocybin—the psychoactive component of magic mushrooms. We show that the homological structure of the brain's functional patterns undergoes a dramatic change post-psilocybin, characterised by the appearance of many transient structures of low stability and of a small number of persistent ones that are not observed in the case of placebo.

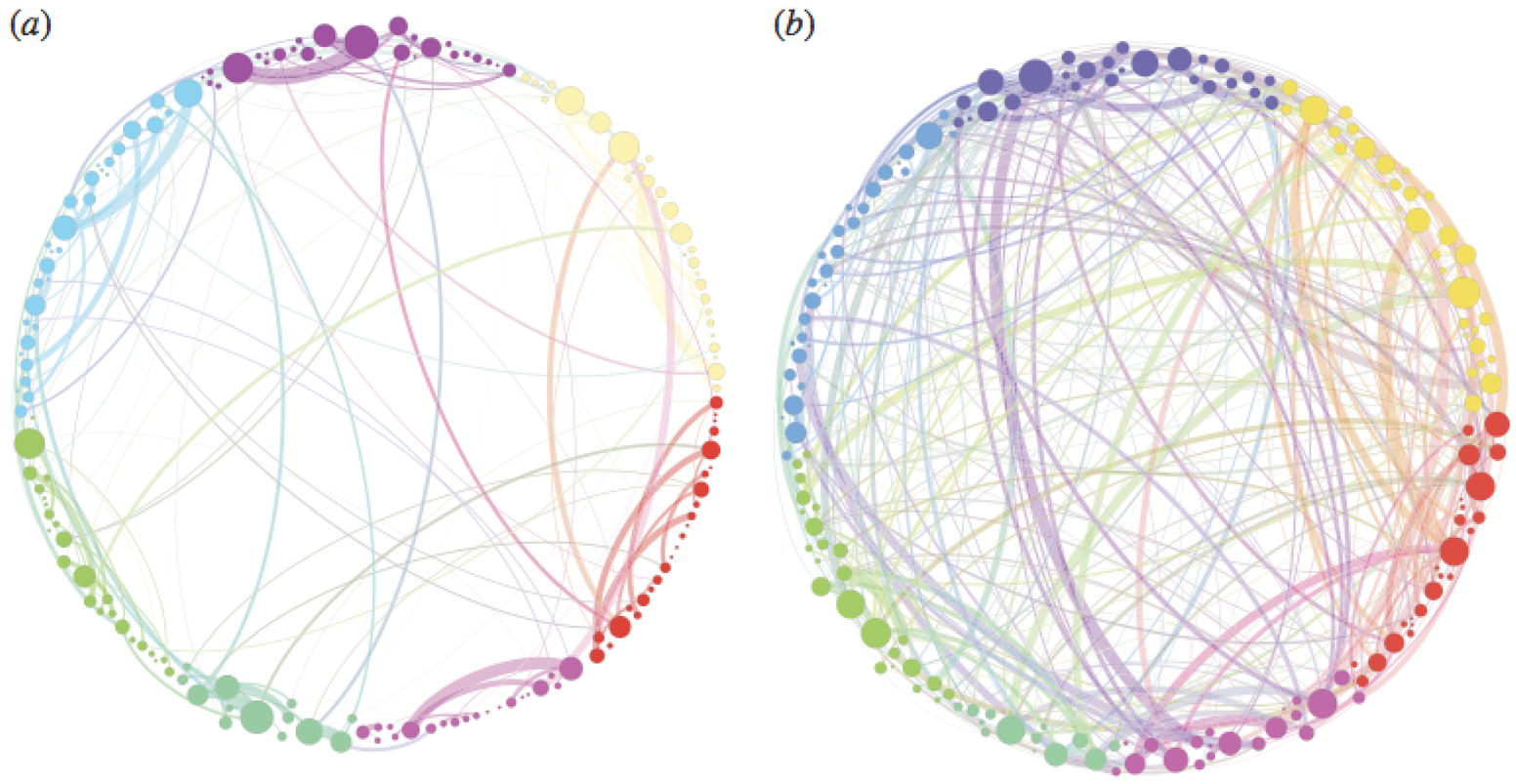


**Figure 1.** Simplified visualisation of the persistence homological scaffolds. The persistence homological scaffolds $H_{pla}$ (a) and $H_{psi}$ (b) are shown for comparison. For ease of visualisation, only the links heavier than 80 (the weight at which the distributions in figure 5a bifurcate) are shown. This value is slightly smaller than the bifurcation point of the weights distributions. In both networks, colours represent communities obtained by modularity optimization on the placebo persistence scaffold using the Louvain method and are used to show the departure of the psilocybin connectivity structure from the placebo baseline.

# How?

## Dataset

5 subjects underwent two resting-state fMRI scans with an injection of a placebo and of psilocybin (the active component of magic mushrooms) halfway through the scan in a balanced manner.

After regression of motion parameters, white matter signal and CSF signal, partial correlations between grey matter regions were produced and analysed .

## Weight Clique Filtration

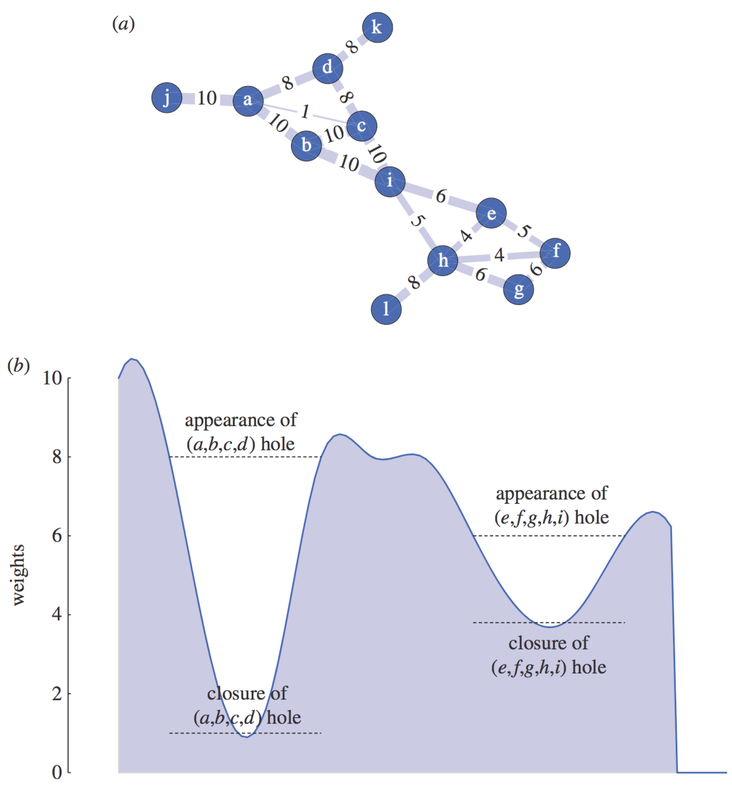


**Figure 2.** The panels display a weighted network (a), its intuitive representation in terms of a stratigraphy in the weight structure according the the weight filtration (b ). By producing a sequence of networks through the filtration, we can study the emergence and relative significance of specific features along the filtration. In this example the hole defined by (a, b, c, d) has a longer persistence (green bars) implying that the boundary of the cycle are much heavier than the internal links that eventually close it. The other hole instead has a much shorter persistence, surviving only for one step and is therefore considered less important in the description of the network homological properties. Note that the births and deaths are defined along the sequence of descending edge weights in the network, not in time.

# Results

## PH results

The placebo group displays generators appearing and persisting over a limited interval of the filtration. On the contrary, most of the generators for the psilocybin group are situated in a well-defined peak at small birth indices, indicating a shorter average cycle persistence. However, the psilocybin distrbution is also endowed with a longer tail implying the existence of a few cycles that are longer lived compared to placebo condition.

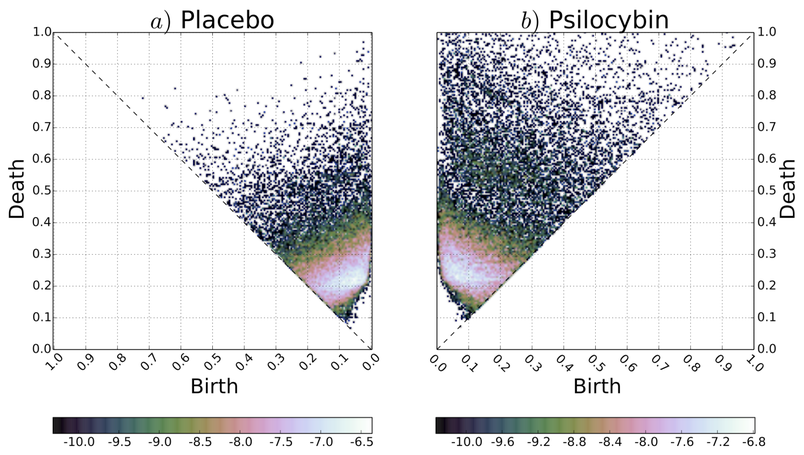


**Figure 3. The probability densities for the $H_1$ generators**. Panel (a) reports the (log-)probability density for the placebo group, whereas panel (b) refers to the psilocybin group. The placebo displays a uniform broad distribution of values for the births–deaths of $H_1$ generators, whereas the plot for the psilocybin condition is very peaked at small values with a fatter tail. These heterogeneities are evident also in the persistence distribution and find explanation in the different functional integration schemes in placebo and drugged brains.

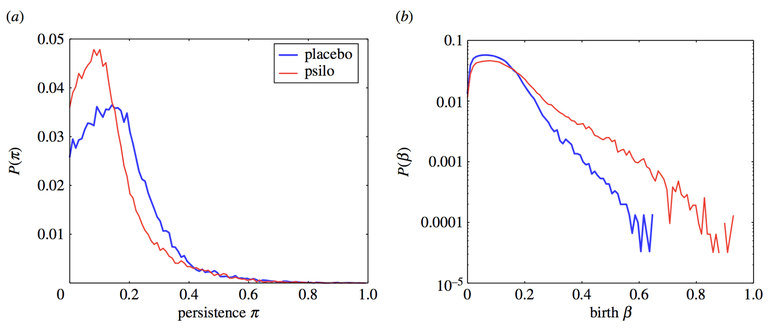


**Figure 4. Comparison of persistence p and birth b distributions.** Panel (a) reports the $H_1$ generators' persistence distributions for the placebo group (blue line) and psilocybin group (red line). Panel (b) reports the distributions of births with the same colour scheme. It is very easy to see that the generators in the psilocybin condition have their persistence peaked at shorter values and a wider range of birth times when compared with the placebo condition.

## Homological Scaffolds

Given a choice of representative cycles (crucial point), we define two networks that summarise the links' information using total persistence and frequency of the cycles:

$$\mathrm{H}_G^p: \quad \omega_e^\pi = \sum_{g_i | e \in g_i} \pi_{g_i}$$

$$\mathrm{H}_G^f: \quad \omega_e^f = \sum_{g_i} 1_{e \in g_i}$$

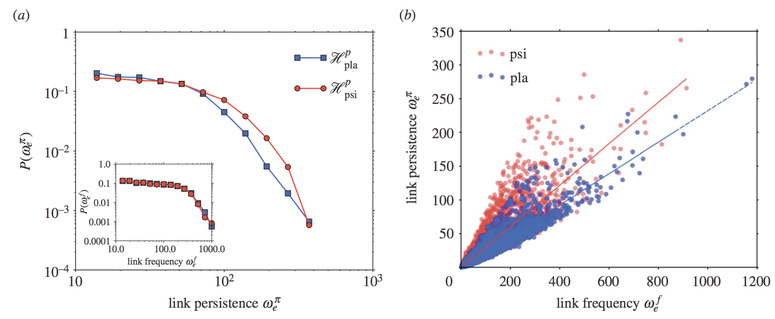


**Figure 5. Statistical features of group homological scaffolds**. Panel (a) reports the (log-binned) probability distributions for the edge weights in the persistence homological scaffolds (main plot) and the frequency homological scaffolds (inset). While the weights in the frequency scaffold are not significantly different, the weight distributions for the persistence scaffold clearly displays a broader tail. Panel (b) shows instead the scatter plot of the edge frequency versus total persistence. In both cases, there is a clear linear relationship between the two, with a larger slope in the psilocybin case. Moreover, the psilocybin scaffold has a larger spread in the frequency and total persistence of individual edges, hinting to a different local functional structure within the functional network of the drugged brains.

# Conclusions

1. There is an increased integration between cortical regions in the psilocybin state.
2. Synaesthesia as possible by-product of greater integration across whole-brain.
3. Scaffold work well, but need refinement.

# Outlook

1. Provide better definition of scaffold (e.g. using Laplacians)
2. Explore data-fusion of different brain imaging technologies.

## References

- Petri G, Expert P, Turkheimer F, Carhart-Harris R, Nutt D, Hellyer PJ, Vaccarino F. (2014) Homological Scaffolds of Brain Functional Networks, J. R. Soc. Interface 11: 20140873. http://dx.doi.org/10.1098/rsif.2014.0873
- Petri G, Scolamiero M, Donato I, Vaccarino F (2013) Topological Strata of Weighted Complex Networks. PLoS ONE 8(6): e66506. doi:10.1371/journal.pone.0066506