Exercises

Masterclass on Classification Problems in Groups and Fusion Systems
Groups of local characteristic p
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 # 1. Let H be a finite group, p a prime and $L = F_p^*(C_H(Y_H))$. Show that

$$Y_L = \Omega_1 Z(O_p(H)).$$

 # 2. Let G be a finite group, p a prime and H a maximal p-local subgroup of G. Put $L = F_p^*(C_H(Y_H))$ and $Q = O_p(H)$. Suppose that H is the unique maximal p-local subgroup of G containing L. Show that

$$Q \trianglelefteq N_G(A)$$

for all $1 \neq A \leq Z(Q)$.

 # 3. Let p be a prime and P be finite p-minimal group of characteristic p with $Z(P) = 1$. Show that

(a) $C_P(Y_P)S \neq P$,

(b) $Y_P = \Omega_1 Z(O_p(P))$, and

(c) $O_p(P)$ is a Sylow p-subgroup of $C_P(Y_P)$.

 # 4. Let p a prime and G a finite group with large p-subgroup Q. Show that one of the following holds:

1. $F^*(G) = O_p(G)$.
2. $F^*(G) = O_{p'}(G) = F(G)$ and $Z(Q)$ is cyclic.
3. G has a simple subnormal subgroup K, K is perfect, p divides K and $F^*(G) = \langle K^Q \rangle$.

 # 5. Let p be prime, q a power of p and n an integer with $n \geq 2$. Let $\mathbb{F}_q$ be a field of order q, V an n-dimensional vector space over $\mathbb{F}_q$ and U a 1-dimensional $\mathbb{F}_q$-subspace of V. Put $G = PSL_{\mathbb{F}_q}(V)$, $\tilde{C} = N_G(U)$, $Q = O_p(\tilde{C})$ and $E = O^p\left(F_p^*(C_{\tilde{C}}(Y_{\tilde{C}}))\right)$. Show that

(a) $\tilde{C}$ is a maximal p-local subgroup of G.

(b) $Q = Y_{\tilde{C}}$.

(c) $E = 1$ and so E is contained in more than one maximal p-local subgroup of G.

(d) Q is a large p-subgroup of G.