(1) A *Goldschmidt grp.* is a simple grp. G whose 2-fusion system is solvable; i.e. if $S \in Syl_p(G)$ then $\exists$ a nontrivial abelian sgp. of S strongly closed in S with respect to G.

Bender gps (Grps. of Lie rank 1 over a field of char 2 $\not\exists$: $L_2(q)$, $Sz(q)$, $U_3(q)$, q even)

Groups with abelian Sylow 2-groups: $L_2(q)$, $q \equiv \pm 3 \bmod 8$, ${}^2G_2(q)$, J_1.

(2) Let $\mathcal{K}$ be the set of known simple groups.

Let $\mathcal{K}_{ev}$ denote the members of $\mathcal{K}$ which are not Goldschmidt grps and not in $Chev^*(p)$, p odd.

(3) Write $\mathcal{K}_{int}$ ("int" = intrinsic) for those quasisimple grps K s.t. $Z(K) \neq 1$ is a 2-grp. and $K/Z(K) \in \mathcal{K}_{ev}$ and $K \not\cong SL_2(q)$, $\hat{A}_7$.

(4) Write $\hat{\mathcal{K}}_{ev} = \mathcal{K}_{ev} \cup \mathcal{K}_{int}$

(5) *Tentative setup*: $\exists$ a saturated f.s. on a 2-gp. S s.t. for $\mathcal{C} \in \underline{C}$, $\mathcal{C}$ is the 2-fusion system of some $\mathcal{K}(\mathcal{C}) \in \mathcal{K}_{ev}$. (Other components "covered" by Walter's thm. and the classical involution theorem)

(6) Partition the problem into several modules.

(7) Define $\mathcal{C} \in \underline{C}$ to be intrinsic if there $\exists$ $t \in Z(\mathcal{C}) \cap \mathcal{I}(\mathcal{C})$. ($\mathcal{C}$ is component in $C_{\mathcal{F}}(t)$)

Note $\mathcal{K}_{int}$ avoids of the grps $\hat{A}_n$ plus a finite number of examples K with $K/Z(K) \in Chev(2)$ sporadic.

(8) Define $\mathcal{C} \in \underline{C}$ to be *subintrinsic* if $\exists$ an involution $t \in \mathcal{C}^f$ and a component $\mathcal{E}$ of $C_{\mathcal{C}}(t)$ s.t. $t \in \mathcal{I}(\mathcal{E}) \cap Z(\mathcal{E})$.

(9) If $\mathcal{C} \in \underline{C}$ and $Z(\mathcal{C}) \neq 1$ then there ex. a comp. in $\underline{C}$ that is intrinsic in $\mathcal{F}$.

(10) If $\mathcal{C} \in \underline{C}$ and $\exists$ an intrinsic member of $\underline{C}(\mathcal{C})$ then $\exists$ an intrinsic member of $\mathcal{F}$.

If $\exists$ a subintrinsic member of $\underline{C}$ then $\exists$ a subintrinsic maximal in $\underline{C}$.

~~Then~~

(12) First module in the partition:

Determine all $\mathcal{F}$ with a subintrinsic member of $\underline{\mathcal{C}}$.

(13) Next setup: Let $\mathcal{K}_{2c} = \mathcal{K}_{ev} \setminus \mathcal{K}_{si}$ where $\mathcal{K}_{si}$ = subintrinsic $\mathcal{K}$'s.

<u>Note</u>: All members of $\mathcal{K}_{2c}$ are simple.

Assume:

(1) Each member of $\underline{\mathcal{C}}$ is the 2-fusion system of some member of $\mathcal{K}_{2c}$.

(2) $\exists$ some involution $t_0 \in \mathcal{F}^f$ s.t. $E(C_{\mathcal{F}}(t)) \neq 1$ and $m_2(C_S(t_0)) = m_2(S)$.

(14) If $\mathcal{F}$ is a saturated 2-fusion system on S and $\mathcal{C}$ is a component of $\mathcal{F}$ in $\mathcal{K}_{2c}$ then $\mathcal{J}(S)$ acts on $\mathcal{C}$.

(15) For $\mathcal{C} \in \underline{\mathcal{C}}$ let $\mathcal{J}(\mathcal{C}) = \{j \in \mathcal{J}(\mathcal{C}) : m_2(C_S(j)) = m_2(S)\}$

$\underline{C}_{\mathcal{J}} = \{\mathcal{C} \in \mathcal{C} : \mathcal{J}(\mathcal{C}) \neq \emptyset\}$

$\mathcal{C} \in \underline{C}_{\mathcal{J}}$ is maximal in $\underline{C}_{\mathcal{J}}$ if $t \in \mathcal{J}(\mathcal{C})$ and $E_4 \cong \langle t, s \rangle \in \mathcal{X}(\mathcal{C})$ and $m_2(C_S(\langle t, s \rangle)) = m_2(S)$, then 12.2.2 does not occur.

<u>Note</u>: If $\mathcal{C}$ is maximal then by (14), $\mathcal{C}$ is a comp. of $C_{\mathcal{F}}(S)$.