Problem session, Equiv. homotopy theory workshop
Copenhagen, Aug 16, 2013
(Lectures by Mike Hill & Stefan Schwede)

Problems:

1. $\underline{\pi}_0 S = \underline{A}$

$\underline{\pi}_1 S = ?$ tom Dieck splitting (& Prieto) give one group at a time
give invariant description. (Aut's of G-sets ?)

$\underline{\pi}_2 S = ?$
etc?

2. Non-equiv. Rational π_* is easy. But
$\mathcal{GH}_{\mathcal{F}in}^{\mathbb{Q}}$ rational global homotopy for finite groups
by Morita theory $\simeq D(\mathcal{GF}_{\mathcal{F}in}^{\mathbb{Q}}) \simeq D(\mathcal{A})$
derived global functors

$$\mathcal{GF}_{\mathcal{F}in}^{\mathbb{Q}} \xrightarrow[\tau]{} \mathrm{Funct}(\mathrm{Epi}^{op}, \mathbb{Q}\text{-Mod}) = \mathcal{A}$$ } has an adjoint integrally

fin. gps & conj cl. of epis

$$F \longmapsto \tau(F) = \left\{ G \longrightarrow F(G)/\text{proper transfers} \right\}$$

What is $\mathrm{Ext}_{\mathcal{A}}(\tau R, \tau R)$ R = Rep'n ring functor
determined on cyclic gps.

~~[illegible]~~ $\underline{A} \longrightarrow \underline{\mathbb{Q}}$ splits one gp at a time,
but not globally.

Questions: What are the complexes of functors in $D(\mathcal{A})$ for images of ku, ku^c, KU, MUP, MU? They will be DGAs.

Rat'l form of Atiyah-Bott-Shapiro orientation, for example.

Rat'l homotopy in $\mathcal{GH}_{\mathcal{F}in}^{\mathbb{Q}} \longleftrightarrow$ homology in $D(\mathcal{A})$

3. (S^1, L) framed w/ left inv. framing

$\downarrow$

η Similarly $(S^3, L) \rightsquigarrow \nu$

And $(S^1,L) \times (S^3,L)$ is 0

So is there an alg 5-manif w/ this as boundary?

η Attach map of $\mathbb{P}^2$ to get from $\mathbb{P}^1$

ν also quaternionic analog

Does Dugger & Isaksen's proof that $\eta\nu = 0$ give any suggestions?

$S^\sigma \xrightarrow{\eta} S^0$ $\quad$ $S^{1+2\sigma} \xrightarrow{\nu} S^0$ $\quad$ for groups larger than C_2,

or universally.

$2\eta = 0 \xrightarrow{\nu_1} \eta\nu = 0$ $\quad$ Can you do this algebraically? (motivically)

$A(C_2) = \mathbb{Z}[t]/t^2-2t$ $\quad t = [C_2/e]$ $\quad \varepsilon = t-1$ $\quad \varepsilon^2 = 1$

or $\quad S^\sigma \wedge S^\sigma \xrightarrow{\varepsilon} S^\sigma \wedge S^\sigma$ twist. $\quad$ $S^V \wedge S^V \underset{\text{twist}}{\xrightarrow{\varepsilon_V}} S^V \wedge S^V$

underly: deg 1

fixed: deg 0

$\varepsilon_V = 1$ if V is complex

$\varepsilon_V^2 = 1$ always

4. Is $A(G)^\times$ gen by $\langle \pm 1, N_H^G(\pm 1) \rangle$? elem abel 2-group known.

rank is not known. tomDieck, Tornehave

Ergun Yalcin & Marcus Szymik know something. Also Jesper G.

counterexample to 1st Question.

Global $gl_1(S)$!!

5. Compute Steenrod algebra for $\underline{M}$

For a Mackey field it is $\mathcal{A} \otimes \mathrm{End}(\underline{M})$ (a student of Gaunce Lewis)

$H\underline{\mathbb{F}_2} \wedge H\underline{\mathbb{F}_2} \simeq \bigvee S^V \wedge H\underline{\mathbb{F}_2}$ over $G = C_2$

"wants to be $\mathcal{A}_*$ but "smeared out".

Every 2 is ρ.

$\underline{M} = A$? Caruso has also looked at related things.

Mike knows some things.

Power ops ? D.L. ops & Normed D.L. ops.

What is this algebra?