

Group Actions - Problems (2013)

Review of 2011 problems

- ① rank 2 finite_gps acts freely on $S^n \times S^m$?
- ②
 - $Qd(p)$ not solved?
 - no equivalent problem to \mathbb{P}^m for $Qd(p)$.
 - does $Qd(p)$ act on a p-local ex $\sim S^n \times S^m$?
 - $p=3$ special? compact group with subgroup $Qd(3)$ (Adem-Grodal)
 - $Qd(p)$ can't act on $S^n \times S^m$ ($n=m$) freely.
 - $H^\infty(Qd(p); \mathbb{Z})$
- ③ generalize Adem-Smith: 2-parameters in $H^*(X; \mathbb{Z})$
- ④ $X \cong S^n$, $G = \Sigma_5$ existence of smooth action?

$$\left\{ \begin{array}{l} G\text{-action with} \\ \text{effective Euler class} \end{array} \right\} \hookrightarrow \left\{ \begin{array}{l} \exists \text{ finite } G\text{-act } X \cong S^n \\ \text{rank 1} \end{array} \right\}$$
- ⑤ Minor condition? Necessary and for
 smooth action: $H \in \mathcal{I}_{\mathbb{Z}_2}(Q)$ must not have $N_G(H)/H$ no finite abelian subgroups.

- ⑥ Solved: Ullin-Yalcin (finite G -act)
 J. Davis (smoothly)
 (Adem) Does every compact Lie grp G act
 freely on some product of spheres. Eg $G = U(n)$

(2)

⑦ wh₂ group \mathbb{L} of finite Vcd act freely

on some $X \simeq S^n \times S^m \times \mathbb{R}^k$?

(what about co-computability?)

⑧ $wh(G) = rk(G)$?

G p-group, rank 3

rank > 3 ?

rank 2 cases still open?

M. de la Kluit (p odd)

\exists smooth actions ~~smooth~~-Yalin

For G finite p-group, \exists finite G -CW X
 \simeq product of spheres, but not smooth actions?

→ no known obstructions

⑨ Carlson question ~~was proven~~ $\sum_{i=0}^{dmX} dmH^i(X; \mathbb{F}_p) \geq 2^r$

$G = (\mathbb{Z}/p)^r \times$ finite, free \mathbb{Q} -act ex.

new cases of Conner conjecture

O.B. Okutan + Yalin: "high dm" cases

⑩ $H^*(B\text{Aut}(X), \mathbb{Q})$ of finite transcendence degree?
 X finite 1-cma. (of Berglund-Madsen)

⑪ G finite act by eff on 2-cx $\Rightarrow X \simeq *$
 Does it have a fixed pt? (Oliver-Sageev)

(3)

II Group Cohomology

- (14) work by Benson - Carlson?
- (17) Benson - Grodal - Henke
 p odd $G_1 \rightarrow G_2$ some p -Sylow Subgroup
 fact ISO on p -blocks \Leftrightarrow iso on mod p column
 varieties
- (21) ask A. Putman
- (23) J. Müller student $G = S_n$ or A_n ?