Flow equivalence of graphs, shifts, and C^*-algebras

18–22 November 2013

Workshop talks

Speaker: Marie-Pierre Béal, Université Paris-Est

Title: Sofic-Dyck shifts

Abstract: We define the class of sofic-Dyck shifts which extends the class of Markov-Dyck shifts introduced by Krieger and Matsumoto. The class of sofic-Dyck shifts is a particular class of shifts of sequences whose finite factors are context-free languages. Sofic-Dyck shifts are sequences of letters over an alphabet which is disjoint union of three sets: the call alphabet, the return alphabet and the internal alphabet. We show that the class of sofic-Dyck shifts corresponds exactly to the class of shifts of sequences whose set of factors is a visibly pushdown language. We define the subclass of finite-type-Dyck shifts and give a decomposition theorem of proper conjugacies. We give an expression of the zeta function of deterministic sofic-Dyck shifts.

This a joint work with Michel Blockelet and Catalin Dima

Speaker: Mike Boyle, University of Maryland

Title: Flow equivalence of sofic shifts

Abstract: This is a progress report on the classification of irreducible sofic shifts up to flow equivalence (FE). This is equivalent to the classification of certain canonical covering factor maps up to flow equivalence. Two maps are flow equivalent if there is a flow equivalence of their domains which takes the quotient relation of one map to the quotient relation of the other.

The multiplicity set of such a map is the set on which it is many to one. We have an extension theorem which reduce the FE classification of almost finite type (AFT) shifts to classifying the restrictions of canonical factor maps to their multiplicity sets. When this restriction is constant to one, the problem reduces to classifying free G-SFTs (SFTs with a shift commuting action by a finite group G) up to G-equivariant flow equivalence. When the G-SFT is mixing, there is a complete algebraic invariant; in general, the problem can be attacked in

an algebraic framework. When the multiplicity set is finite, there are complete invariants.

For irreducible strictly sofic shifts with a Fischer cover by a map at most 2-to-one, we prove the FE-classification reduces to the classification of $\mathbb{Z}_2$-SFTs up to $\mathbb{Z}_2$-equivariant flow equivalence. In this general "2-sofic" case (in contrast to the AFT case), the multiplicity set of the factor map can be dense.

This is joint work with Toke Carlsen and Søren Eilers.

Speaker: Toke Carlsen, Norwegian University of Science and Technology

Title: Partial actions and KMS states on relative graph C^*-algebras

Abstract: In this talk I will tell about the preprint "Partial actions and KMS states on relative graph C^*-algebras" (arXiv:1311.0912) which I have recently written together with Nadia Larsen from Oslo. Relative graph C^*-algebras were introduced by Muhly and Tomforde as generalizations of both graph C^*-algebras and their Toplitz extensions. In our paper, Nadia and I realize relative graph C^*-algebras as partial crossed products and use this to characterize the KMS states of certain actions on arbitrary relative graph C^*-algebras. We thereby obtain a complete concrete description of the convex set of all KMS states for a big class of graphs which includes all finite graphs.

Speaker: Alfredo Costa, University of Coimbra

Title: Applications of the Karoubi envelope of a shift

Abstract: In this talk we deduce flow equivalence invariants of shifts — some new and some old — from the flow invariance of the Karoubi envelope of the shift and of its action on the Krieger cover. The applications will be illustrated with concrete examples, with an emphasis on the sofic case. This is joint work with Benjamin Steinberg.

Speaker: Søren Eilers, University of Copenhagen

Title: An atlas of graph C^*-algebras for small graphs

Abstract: Ever since the inception of the Cuntz-Krieger algebras, there has been considerable interest in trying to decide which SFTs yield C^*-algebras which are isomorphic or Morita equivalent. One approach to this question, which was initiated by Enomoto, Fujii and Watatani as early as 1981, is to provide a complete catalogue of all the C^*-algebras associated to SFTs associated to simple graphs with N vertices, for N ranging somewhere in $\{2, 3, 4\}$. The advent of more and more sophisticated classification results has of course allowed better and better such atlases, but it is only with the recent work reported on in Ruiz' talk that

we are able to fully deal with all graphs in this range (there are 3044 different such graphs for $N = 4$). Our work illustrates the nature of the invariants used, emphasizes the distinction between flow equivalence and isomorphism, and have bearing on the conjectures of Abrams and Tomforde to the effect that (stable) isomorphism of graph C^*-algebras is equivanet to isomorphism of the Leavitt path algebras. This is joint work with Efren Ruiz and Adam Sørensen, as well as Rune Johansen.

Speaker: Rune Johansen, University of Copenhagen

Title: Towards a flow classification of sofic beta-shifts

Abstract: For each $\beta > 1$, one can define a so called beta-shift. These shift spaces give an interesting link between number theory and symbolic dynamics and their internal structure make them useful as a test case in a number of situations. The aim of the this work has been to classify sofic beta-sthifts up to flow equivalence. Given a sofic beta-shift, the Fischer cover is constructed, and the Bowen-Franks invariant of the corresponding shift of finite type is computed. It is shown that the covering map is 2 to 1, and this allows the construction of a fiber product cover with a natural action of $\mathbb{Z}/2\mathbb{Z}$. An explicit construction is given to classify the corresponding shifts of finite type up to flow equivalence. These constructions result in a number of flow invariants which all depend only on a single number which is easily computed from β. It is hoped that recent work by Boyle, Carlsen and Eilers will make it possible to show that this number is a complete invariant for flow equivalence of sofic beta-shifts.

Speaker: Wolfgang Krieger, University of Heidelberg

Title: Property (A) of subshifts and flow equivalence

Abstract: It is shown that Property (A) of subshifts, and the semigroup that is invariantly associated to subshifts with Property (A), are invariants of flow-equivalence. This is applied to R-graph shifts.

Speaker: Kengo Matsumoto, Yokohama City University

Title: Orbit equivalence, full groups and Cuntz-Krieger algebras

Abstract: In this talk, I would like to explain that there exist strong relationships among the continuous orbit equivalence classes of one-sided topological Markov shifts, the isomorphism classes of continuous full groups, and the isomorphism classes of Cuntz-Krieger algebras. Main part of the talk is due to a recent joint paper with Hiroki Matui "Continuous orbit equivalence of topological Markov shifts and Cuntz-Krieger algebras, ArXiv:1307.1299 ".

Speaker: David Pask, University of Wollongong

Title: Flow equivalence and graph C^*-algebras

Abstract: Shifts of finite type are, up to conjugacy, the edge shifts of directed graphs. The relations defining a graph C^*-algebra encode the connectivity of a directed graph in terms of operators on a Hilbert space. Closer examination of the structure of a graph C^*-algebra reveals deep connections between its internal properties and the dynamical properties of the associated shift of finite type. The first part of the talk will be an introduction to graph C^*-algebras.

It has been known for some time now that flow equivalence on shifts of finite type are related to certain graphical constructions, called splittings and delays, on the underlying directed graph. In the second part of this talk we will examine how the operations of splittings and delays affect the associated graph C^*-algebras. This is joint work with Teresa Bates.

Speaker: Gunnar Restorff, University of the Faroe Islands

Title: Classification of purely infinite graph algebras with at most four primitive ideals

Abstract: In the years following Kirchberg-Phillips' classification of purely infinite, simple, nuclear, separable C^*-algebras in the bootstrap class, many people have tried to generalize these results to non-simple C^*-algebras. One main tool has been Kirchberg's result concerning lifting of invertible ideal related KK-elements to isomorphisms. The work of Meyer, Nest, Bentmann and Köhler showed some positive results and some negative results for classification of purely infinite algebras with finite ideal lattices. Generally, it seems that much of what can go wrong, already goes wrong for the case of four primitive ideals.

In contrast to this, there is a classification of purely infinite Cuntz-Krieger algebras up to stable isomorphism for any (finite) ideal lattice. Thus it is natural to ask the question, why we do have classification for these algebras but not in general. In this talk we will give a quite complete answer of all the most natural questions that come up in connection with the classification problem of purely infinite Cuntz-Krieger algebras - and more generally, graph algebras - with at most four primitive ideals.

This is joint work with Sara Arklint and Efren Ruiz.

Speaker: Efren Ruiz, University of Hawai'i at Hilo

Title: Classification of graph C^*-algebras over finite graphs

Abstract: In 2004, Gunnar Restorff completed the classification of Cuntz-Krieger algebra whose adjacency matrix satisfy Condition (II) of Cuntz using an invariant called reduced filtered K-theory. Using the fact that every Cuntz-Krieger algebra is isomorphic to a graph C^*-algebra over a finite graph with no sinks and sources,

one can then reformulate Restorff's classification as follows: two Cuntz-Krieger algebras with finitely many ideals $C^*(E)$ and $C^*(F)$ are Morita equivalent if and only if there is a finite sequence of graph moves generated by the "Franks Moves" and a move called the "Cuntz-Splice" to get from E to F. In this talk, I will discuss my joint work with Søren Eilers and Adam Sørensen in which we generalize Restorff's classification via moves on finite graphs. In particular, our graphs are allowed to have sinks and our C^*-algebras are allowed to have infinitely many ideals. We can use our results to classify the so-called quantum lens spaces which are C^*-algebras associated to certain countable shift spaces of finite type.

Speaker: Benjamin Steinberg, The City College of New York

Title: The Karoubi envelope and the classification of Markov Dyck shifts

Abstract: In this talk we introduce the Karoubi envelope of a shift space, which is a small category that is a flow equivalence invariant up to natural equivalence. We sketch a simplified proof for the special case of a sofic shift using results of Nasu. We end by classifying Markov-Dyck shifts up to flow equivalence under fairly mild hypotheses on the graphs.

Speaker: Mark Tomforde, University of Houston

Title: Flow equivalence and its uses in the classification of C^*-algebras and algebras

Abstract: The notion of flow equivalence for shifts of finite type was one of the key ingredients in the classification of the simple Cuntz-Krieger algebras, and it has similarly been used to obtain partial classification results for simple Leavitt path algebras of finite graphs. In this talk I will discuss how flow equivalence has recently inspired ideas that have been used to classify unital graph C^*-algebras without appealing to the strength of the Kirchberg-Phillips Classification Theorem, and have also been used to give a complete classification of Leavitt path algebras of infinite graphs with a finite number of vertices. I'll conclude with a discussion of how flow equivalence is related to the missing pieces needed for a complete classification of simple Leavitt path algebras of finite graphs.

Additional Talks

Speaker: Rasmus Bentmann, Georg-August-Universität Göttingen

Title: Contributions to the structure theory of non-simple C^*-algebras (Ph.D. Defence)

Abstract: This thesis is mainly concerned with classification results for non-simple purely infinite C^*-algebras, specifically Cuntz-Krieger and graph C^*-algebras, and continuous fields of Kirchberg algebras. We perform some computations concerning projective dimension in filtrated K-theory. In joint work with Sara Arklint and Takeshi Katsura, we provide a range result complementing Gunnar Restorff's classification theorem for Cuntz-Krieger algebras and we investigate reduction of filtrated K-theory for C^*-algebras of real rank zero, thereby obtaining a characterization of Cuntz-Krieger algebras with primitive ideal space of accordion type. We establish a universal coefficient theorem computing Eberhard Kirchberg's ideal-related KK-groups over a finite space for algebras with vanishing boundary maps. This result is used to classify certain continuous fields of Kirchberg algebras. A stronger result for one-parameter continuous fields is obtained in joint work with Marius Dadarlat. Finally, we compute Stefan Schwede's n-order for certain triangulated categories of C^*-algebras.

Speaker: Mike Boyle, University of Maryland

Title: What is flow equivalence of shifts of finite type and sofic shifts (and why)?

Abstract: Talk for the general audience associated with the Centre for Symmetry and Deformation. Lunch will be served.

Speaker: Harald Helfgott, CNRS/ENS

Title: The ternary Goldbach problem: history and the circle method

Abstract: The ternary Goldbach conjecture (1742) asserts that every odd number greater than 5 can be written as the sum of three prime numbers. Following the pioneering work of Hardy and Littlewood, Vinogradov proved (1937) that every odd number larger than a constant C satisfies the conjecture. In the years since then, there has been a succession of results reducing C, but only to levels much too high for a verification by computer up to C to be possible ($C > 10^{1300}$). (Works by Ramare and Tao have solved the corresponding problems for six and five prime numbers instead of three.)

My recent work proves the conjecture.

This first talk will cover the history of the problem, the general framework of Fourier-analytic approaches to arithmetical problems (the circle problem) and the challenges posed by such approaches.

List of participants

1. Marie-Pierre Béal, Université Paris-Est
2. Mike Boyle, University of Maryland
3. Toke Carlsen, Norwegian University of Science and Technology
4. Alfredo Costa, University of Coimbra
5. Rune Johansen, University of Copenhagen
6. Wolfgang Krieger, University of Heidelberg
7. Kengo Matsumoto, Yokohama City University
8. David Pask, University of Wollongong
9. Gunnar Restorff, University of the Faroe Islands
10. Efren Ruiz, University of Hawai'i at Hilo
11. Benjamin Steinberg, The City College of New York
12. Mark Tomforde, University of Houston
13. Søren Eilers, University of Copenhagen
14. Sanaz Pooya, Lund University
15. Søren Knudby, University of Copenhagen
16. Dominic Enders, University of Copenhagen
17. Tyrone Crisp, University of Copenhagen
18. Martin Wedel Jacobsen, University of Copenhagen
19. Rasmus Bentmann, Georg-August-Universität Göttingen
20. Mauricio Gomez, University of Copenhagen
21. Sara Arklint, University of Copenhagen
22. Jamie Gabe, University of Copenhagen
23. Michel Hilsum, IMJ-PRG
24. Jeong Hee Hong, Korea Maritime University
25. Wojciech Szymanski, University of Southern Denmark