LECTURES ON REPRESENTATION THEORY OF REDUCTIVE p-ADIC GROUPS.

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1. Abstract

For a reductive p-adic group G we give a description of the category $\mathcal{M}(G)$ of smooth representations of G and describe the structure of the set $\mathrm{Irr}(G)$ of isomorphism classes of irreducible smooth representations.

The answer we will provide is in term of an algebraic variety $\Omega(G)$ that is a union of (infinite number) of connected components Ω. Each component Ω is an affine algebraic variety isomorphic to a quotient of a torus by the action of a finite group. These components are parameterized by irreducible cuspidal representations of Levi subgroups of G modulo some equivalence.

We will construct a natural map $\mathbf{inf} : \mathrm{Irr}(G) \to \Omega(G)$ (infinitesimal caracter). This map is a finite to one epimorphism that is a bijection on an open dense subset $\Omega(G)^{irr} \subset \Omega(G)$.

We will show that the category $\mathcal{M}(G)$ naturally decomposes as a direct product of subcategories $\mathcal{M}(\Omega)$ corresponding to components Ω of $\Omega(G)$ and that each category $\mathcal{M}(\Omega)$ is equivalent to the category of modules over some unital algebra $H(\Omega)$.

We will also indicate a characterization of the variety $\Omega(G)$ as the spectrum of so-called Bernstein center of the category $\mathcal{M}(G)$.

A key tool in proving these results will be the second adjointness theorem, a certain adjunction between the induction functors and the Jacquet functors.

The presentation will be based on Bernstein's Harvard notes. We will provide many exercises to illustrate basic ideas (including exercise sheets).

2. Lectures

- Lecture Ia. ℓ-groups and their smooth representations.
- Lecture Ib. Compact groups and compact representations.
- Lecture IIa. Standard functors in categories of representations.
- Lecture IIb. Splitting off of cuspidal part.
- Lecture IIIa. Geometric lemma and its corollaries.
- Lecture IIIb. Central algebra of a reductive p-adic group. Second adjointness theorem.
- Lecture IVa. Geometric structures in representation theory.
- Lecture IVb. Complements: proofs of basic results.