Local Langlands correspondence and examples of ABPS conjecture

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Notation

- F non-archimedean local field : finite extension of $\mathbb{Q}_p$ or $\mathbb{F}_p((t))$
- $\mathcal{O}_F = \{x \in F, v(x) \geqslant 0\}$, ring of integers of F
- $\mathfrak{p}_F = \{x \in F, v(x) > 0\}$, unique maximal ideal of $\mathcal{O}_F$
- $k_F = \mathcal{O}_F / \mathfrak{p}_F \simeq \mathbb{F}_q$, residual field of F (finite field)
- $\varpi_F \in \mathcal{O}_F$ uniformizer of F
- $\overline{F}$ separable algebraic closure of F

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$\forall n \geqslant 1, \exists! \ F \subset F_n \subset \overline{F}$ unramified extension of degree n
F_n/F is Galois, $k_{F_n} \simeq \mathbb{F}_{q^n}$ and $Gal(F_n/F) \xrightarrow{\sim} Gal(k_{F_n}/k_F)$

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$Gal(F_{ur}/F) \simeq \varprojlim_{n\geqslant 1} \mathbb{Z}/n\mathbb{Z} = \widehat{\mathbb{Z}}$
$\Phi_F \in Gal(F_{ur}/F),\ \Phi_{F|F_n} = \Phi_n$

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Topology on Γ_F : Open neighborhood basis at the identity are $Gal(\overline{F}/K)$ with K/F finite extension
Γ_F is a compact Haussdorf profinite group

$$Gal(\overline{F}/F) \longrightarrow Gal(F_{ur}/F) \longrightarrow 1$$

$$1 \longrightarrow I_F \longrightarrow Gal(\overline{F}/F) \longrightarrow Gal(F_{ur}/F) \longrightarrow 1$$

$$
\begin{array}{ccccccccc}
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 & & & & & & \uparrow & & \\
 & & & & & & < \Phi > & &
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Definition

The Weil group of F is the topological group, with underlying abstract group the inverse image in Γ_F of $< \Phi >$, such that :

- I_F is an open subgroup of W_F
- the topology on I_F, as subspace of W_F, coincides with its natural topology as subspace of Γ_F

Let $W_F^{der} = \overline{[W_F, W_F]}$ and $W_F^{ab} = W_F / W_F^{der}$.

Artin Reciprocity Map

There is a canonical continuous group morphism

$$a_F : W_F \longrightarrow F^{\times}$$

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- $a_F(I_F) = \mathcal{O}_F$

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If T is a split torus over F, then

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(+ equality of L-functions)
This is the local Langlands correspondence for $GL(1)$ and for split tori.

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Godement and Jacquet defined L-functions for the representations of $GL_n(F)$. For $n = 2$,

- for supercuspidal representations are 1
- for Steinberg representation (and their twist by character) involve one factor with q^{-s}
- for other involve two factors with q^{-s}

Definition

The Weil-Deligne group is $WD_F = \mathbb{C} \rtimes W_F$ where W_F acts on $\mathbb{C}$ by $wxw^{-1} = |w|x$ for $w \in W_F, x \in \mathbb{C}$.

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A Langlands parameter for $GL(n)$ is a continous morphism $\psi : WD_F \longrightarrow GL_n(\mathbb{C})$ such that the restriction to $\mathbb{C}$ is algebraic, the image of $\mathbb{C}$ consists of unipotent elements and the image of W_F consists of semisimple elements.

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We can identify one Langlands parameter ψ to a pair (ρ, N) where $\rho : W_F \longrightarrow GL_n(\mathbb{C})$ is a continous morphism such that his image consists of semisimple elements and $N \in \mathfrak{gl}_n(\mathbb{C})$ is such that $\rho(w)N\rho(w)^{-1} = |w|N$ for all $w \in W_F$.

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$$\psi(z, w) = exp(zN)\rho(w), \ \ (z, w) \in WD_F$$

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This group is isomorphic to $SL_2(\mathbb{C}) \times W_F$ via the map $(x, w) \mapsto (x h_w, w)$.

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Local Langlands correspondence (conjecture)

Let G a split connected reductive group over F and $\widehat{G}$ is Langlands dual. Let $\Pi(G)$ the set of isomorphism classes of irreducible representations of G and $\Phi(G)$ the set of equivalence classes of admissible morphisms for G. There is finite-to-one map $rec : \Pi(G) \longrightarrow \Phi(G)$ with these properties

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- $\pi \in \Pi(G)$ is essentially square-integrable, if and only if, the image of W_F' by $\phi_\pi : W_F' \longrightarrow \widehat{G}$ is not contained in any proper Levi subgroup of $\widehat{G}$.
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- $\pi \in \Pi(G)$, χ a character of $F^\times$, then $rec(\pi \otimes \chi) = rec(\pi)\chi$.
- Equality of L-functions and ε-factors.

Let $J = \begin{pmatrix} & & 1 \\ & \cdot^{\cdot^{\cdot}} & \\ 1 & & \end{pmatrix}$ and $J' = \begin{pmatrix} & & & 1 \\ & & 1 & \\ & -1 & & \\ -1 & & & \end{pmatrix}$.

For $M \in GL_4(F)$, ${}^{\tau}M = J^{t}MJ$.

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There are three parabolic subgroups of $Sp_4(F)$: B, P, Q with respective Lévi T, M, L.

- $T = \left\{ \begin{pmatrix} t_1 & & & \\ & t_2 & & \\ & & t_2^{-1} & \\ & & & t_1^{-1} \end{pmatrix}, t_i \in F^\times \right\}, \; T \simeq (F^\times)^2$

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- $M = \left\{ \begin{pmatrix} g & \\ & {}^\tau g^{-1} \end{pmatrix}, g \in GL_2(F) \right\},\ M \simeq GL_2(F)$
- $L = \left\{ \begin{pmatrix} t & & \\ & g & \\ & & t^{-1} \end{pmatrix}, t \in F^\times, g \in SL_2(F) \right\},\ L \simeq GL_1(F) \times SL_2(F)$

The Weyl group of G is $W = N_G(T)/T \simeq \mathfrak{S}_2 \ltimes (\mathbb{Z}/2\mathbb{Z})^2$, generated by

$$a = \begin{pmatrix} & 1 & & \\ -1 & & & \\ & & & -1 \\ & & 1 & \end{pmatrix}, b = \begin{pmatrix} 1 & & & \\ & & 1 & \\ & -1 & & \\ & & & 1 \end{pmatrix}$$

Action of W on T and on their irreducible representations

w	$w \cdot (t_1, t_2)$	$w \cdot (\chi_1 \boxtimes \chi_2)$
e	(t_1, t_2)	$\chi_1 \boxtimes \chi_2$
a	(t_2, t_1)	$\chi_2 \boxtimes \chi_1$
b	(t_1, t_2^{-1})	$\chi_1 \boxtimes \chi_2^{-1}$
ab	(t_2^{-1}, t_1)	$\chi_2 \boxtimes \chi_1^{-1}$
ba	(t_2, t_1^{-1})	$\chi_2^{-1} \boxtimes \chi_1$
aba	(t_1^{-1}, t_2)	$\chi_1^{-1} \boxtimes \chi_2$
bab	(t_2^{-1}, t_1^{-1})	$\chi_2^{-1} \boxtimes \chi_1^{-1}$
$abab$	(t_1^{-1}, t_2^{-1})	$\chi_1^{-1} \boxtimes \chi_2^{-1}$

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We consider two inertial classes $\mathfrak{s} = [T, \chi \boxtimes 1]$ and $\mathfrak{t} = [T, \chi \boxtimes \zeta]$.

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The isotropy subgroup of $\mathfrak{s}$ is the subgroup of elements w in W such that there exists an unramified character $\phi \in \psi(T)$, $w \cdot (\chi \boxtimes 1) = (\chi \boxtimes 1) \otimes \phi$.

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For $\mathfrak{s}$ and $\mathfrak{t}$ we have $W_\mathfrak{s} = W_\mathfrak{t} = \{1, b\}$ and $T_\mathfrak{s} = T_\mathfrak{t} = \psi(T) \simeq (\mathbb{C}^\times)^2$.

The extended quotient is

$$T_{\mathfrak{s}} /\!/ W_{\mathfrak{s}} = T_{\mathfrak{s}} / W_{\mathfrak{s}} \sqcup T_{\mathfrak{s}}^{b} / Z_{W_{\mathfrak{s}}}(b)$$

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$b \cdot (t_1, t_2) = (t_1, t_2) \iff (t_1, t_2^{-1}) = (t_1, t_2) \iff t_2 = \pm 1.$

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Remark : It is the same extended quotient for $\mathfrak{s}$ and $\mathfrak{t}$

Theorem (Sally, Tadic)

Let ψ_1, ψ_2 two characters of $F^\times$. Then $\psi_1 \times \psi_2 \rtimes 1$ is reducible, if and only if

- $\psi_1^{\pm 1} = \nu^{\pm 1}\psi_2$
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$\psi \times \nu \rtimes 1$ has two subquotients : $\psi \rtimes 1_{SL_2}$ and $\psi \rtimes St_{SL_2}$

$\mathfrak{s} = [T, \chi \boxtimes 1]$

point of $T_{\mathfrak{s}} /\!/ W_{\mathfrak{s}}$	cocharacter	constituant	representation
$(e, (z_1, z_2))$		$\psi_1\chi \times \psi_2 \ltimes 1$	$\psi_1\chi \times \psi_2 \ltimes 1$
$(e, (z, q^{-1}))$		$\psi\chi \rtimes 1_{SL_2}$	$\psi\chi \times \upsilon \ltimes 1$
$(b, (z, 1))$		$\psi\chi \rtimes St_{SL_2}$	
$(e, (z, -1))$		$\psi\chi \rtimes T_1^{\varepsilon}$	$\psi\chi \times \varepsilon \ltimes 1$
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$(e, (z, q^{-1}))$	$(1, 1)$	$\psi\chi \rtimes 1_{SL_2}$	$\psi\chi \times \nu \ltimes 1$
$(b, (z, 1))$	$(1, \tau^{-2})$	$\psi\chi \rtimes St_{SL_2}$	
$(e, (z, -1))$	$(1, 1)$	$\psi\chi \rtimes T_1^\varepsilon$	$\psi\chi \times \varepsilon \ltimes 1$
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$(e,(z_1,z_2))$	$(1,1)$	$\psi_1\chi \times \psi_2 \ltimes 1$	$\psi_1\chi \times \psi_2 \ltimes 1$
$(e,(z,q^{-1}))$	$(1,1)$	$\psi\chi \rtimes 1_{SL_2}$	$\psi\chi \times \upsilon \ltimes 1$
$(b,(z,1))$	$(1,\tau^{-2})$	$\psi\chi \rtimes St_{SL_2}$	
$(e,(z,-1))$	$(1,1)$	$\psi\chi \rtimes T_1^{\varepsilon}$	$\psi\chi \times \varepsilon \ltimes 1$
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$$(T_{\mathfrak{s}} /\!/ W_{\mathfrak{s}})_{u_e} = T_{\mathfrak{s}}^{b+}/W_{\mathfrak{s}}, \text{ cocharacter } h_{u_e}(\tau) = (1,\tau^{-2})$$

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$(e, (z_1, z_2))$	$(1, 1)$	$\psi_1\chi \times \psi_2 \ltimes 1$	$\psi_1\chi \times \psi_2 \ltimes 1$
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Remark : $(e, (z, -1))$ and $(b, (z, -1))$ are in the same L-packet

$\mathfrak{t} = [T, \chi \boxtimes \zeta]$

point of $T_{\mathfrak{t}} /\!/ W_{\mathfrak{t}}$	cocharacter	constituant	representation
$(e, (z_1, z_2))$		$\psi_1\chi \times \psi_2 \ltimes 1$	$\psi_1\chi \times \psi_2 \ltimes 1$
$(e, (z, 1))$		$\psi\chi \rtimes T_1^{\zeta}$	$\psi\chi \times \zeta \ltimes 1$
$(b, (z, 1))$		$\psi\chi \rtimes T_2^{\zeta}$	
$(e, (z, -1))$		$\psi\chi \rtimes T_1^{\varepsilon\zeta}$	$\psi\chi \times \varepsilon\zeta \ltimes 1$
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$(e, (z, -1))$	$(1, 1)$	$\psi\chi \rtimes T_1^{\varepsilon\zeta}$	$\psi\chi \times \varepsilon\zeta \ltimes 1$
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$(e, (z_1, z_2))$	$(1, 1)$	$\psi_1\chi \times \psi_2 \ltimes 1$	$\psi_1\chi \times \psi_2 \ltimes 1$
$(e, (z, 1))$	$(1, 1)$	$\psi\chi \rtimes T_1^{\zeta}$	$\psi\chi \times \zeta \ltimes 1$
$(b, (z, 1))$	$(1, 1)$	$\psi\chi \rtimes T_2^{\zeta}$	
$(e, (z, -1))$	$(1, 1)$	$\psi\chi \rtimes T_1^{\varepsilon\zeta}$	$\psi\chi \times \varepsilon\zeta \ltimes 1$
$(b, (z, -1))$		$\psi\chi \rtimes T_2^{\varepsilon\zeta}$	

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$(e, (z, -1))$	$(1, 1)$	$\psi\chi \rtimes T_1^{\varepsilon\zeta}$	$\psi\chi \times \varepsilon\zeta \ltimes 1$
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$\mathfrak{t} = [T, \chi \boxtimes \zeta]$

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$(e,(z_1,z_2))$	$(1,1)$	$\psi_1\chi \times \psi_2 \ltimes 1$	$\psi_1\chi \times \psi_2 \ltimes 1$
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$(b, (z, -1))$	$(1, 1)$	$\psi\chi \rtimes T_2^{\varepsilon\zeta}$	

$$T_{\mathfrak{t}} /\!/ W_{\mathfrak{t}} = (T_{\mathfrak{t}} /\!/ W_{\mathfrak{t}})_{u_0} \sqcup (T_{\mathfrak{t}} /\!/ W_{\mathfrak{t}})_{u_e}$$

$$(T_{\mathfrak{t}} /\!/ W_{\mathfrak{t}})_{u_0} = T_{\mathfrak{s}} / W_{\mathfrak{s}} \sqcup T_{\mathfrak{s}}^{b} / W_{\mathfrak{s}}, \text{ cocharacter } h_{u_0}(\tau) = (1, 1)$$

$$(T_{\mathfrak{t}} /\!/ W_{\mathfrak{t}})_{u_e} = \emptyset$$

$\mathfrak{t} = [T, \chi \boxtimes \zeta]$

point of $T_{\mathfrak{t}} /\!/ W_{\mathfrak{t}}$	cocharacter	constituant	representation
$(e,(z_1,z_2))$	$(1,1)$	$\psi_1\chi \times \psi_2 \ltimes 1$	$\psi_1\chi \times \psi_2 \ltimes 1$
$(e,(z,1))$	$(1,1)$	$\psi\chi \rtimes T_1^{\zeta}$	$\psi\chi \times \zeta \ltimes 1$
$(b,(z,1))$	$(1,1)$	$\psi\chi \rtimes T_2^{\zeta}$	
$(e,(z,-1))$	$(1,1)$	$\psi\chi \rtimes T_1^{\varepsilon\zeta}$	$\psi\chi \times \varepsilon\zeta \ltimes 1$
$(b,(z,-1))$	$(1,1)$	$\psi\chi \rtimes T_2^{\varepsilon\zeta}$	

$$T_{\mathfrak{t}} /\!/ W_{\mathfrak{t}} = (T_{\mathfrak{t}} /\!/ W_{\mathfrak{t}})_{u_0} \sqcup (T_{\mathfrak{t}} /\!/ W_{\mathfrak{t}})_{u_e}$$

$$(T_{\mathfrak{t}} /\!/ W_{\mathfrak{t}})_{u_0} = T_{\mathfrak{s}}/W_{\mathfrak{s}} \sqcup T_{\mathfrak{s}}^{b}/W_{\mathfrak{s}}, \text{ cocharacter } h_{u_0}(\tau) = (1,1)$$

$$(T_{\mathfrak{t}} /\!/ W_{\mathfrak{t}})_{u_e} = \emptyset$$

Remark : $(e,(z,-1))$ and $(b,(z,-1))$ are in the same L-packet, for $(e,(z,1))$ and $(b,(z,1))$ is the same

The Langlands dual group of $Sp_4(F)$ is $SO_5(\mathbb{C})$, with $SO_5(\mathbb{C}) = \{g \in SL_5(\mathbb{C}), {}^t gJg = J\}$ and the maximal torus

$$\widehat{T} = \left\{ \begin{pmatrix} z_1 & & & & \\ & z_2 & & & \\ & & 1 & & \\ & & & z_2^{-1} & \\ & & & & z_1^{-1} \end{pmatrix}, z_i \in \mathbb{C}^\times \right\} \simeq (\mathbb{C}^\times)^2$$

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The Lie algebra of $SO_5(\mathbb{C})$ is

$$\mathfrak{so}_5(\mathbb{C}) = \left\{ \begin{pmatrix} y_1 & x_{12} & x_{13} & x_{14} & 0 \\ x_{21} & y_2 & x_{23} & 0 & -x_{14} \\ x_{31} & x_{32} & 0 & -x_{23} & -x_{13} \\ x_{41} & 0 & -x_{32} & -y_2 & -x_{12} \\ 0 & -x_{41} & -x_{31} & -x_{21} & -y_1 \end{pmatrix}, x_{ij}, y_i \in \mathbb{C} \right\}$$

The Langlands parameters for $\chi_1 \boxtimes \chi_2$ is

$$\begin{array}{rccc} \rho(\chi_1, \chi_2): & W_F & \longrightarrow & \widehat{T} \\ & w & \longmapsto & \begin{pmatrix} \chi_1(w) & & & & \\ & \chi_2(w) & & & \\ & & 1 & & \\ & & & \chi_2(w)^{-1} & \\ & & & & \chi_1(w)^{-1} \end{pmatrix} \end{array}$$

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The Langlands parameters for the irreducible subquotients of $\psi_1\chi \times \psi_2 \rtimes 1$ are $(\rho(\psi_1\chi, \psi_2), 0)$, $(\rho(\psi_1\chi, \upsilon), 0)$ and $(\rho(\psi_1\chi, \upsilon), N)$ with

$$N = \begin{pmatrix} 0 & & & & \\ & 0 & 1 & & \\ & & 0 & -1 & \\ & & & 0 & \\ & & & & 0 \end{pmatrix}$$

The Langlands parameters are also the morphisms from W_F' to $SO_5(\mathbb{C})$. For $(\rho(\psi_1\chi, \nu), N)$ the corresponding morphism is

$$\begin{array}{rccl} \phi: & W_F' & \longrightarrow & \widehat{T} \\ & (w,s) & \longmapsto & \begin{pmatrix} (\psi_1\chi)(w) & & \\ & S_3(s) & \\ & & (\psi_1\chi)(w)^{-1} \end{pmatrix} \end{array}$$

where S_3 is the irreducible representation of dimension 3 of $SL_2(\mathbb{C})$.

The Langlands parameters are also the morphisms from W'_F to $SO_5(\mathbb{C})$. For $(\rho(\psi_1\chi, v), N)$ the corresponding morphism is

$$\begin{array}{rccc} \phi: & W'_F & \longrightarrow & \widehat{T} \\ & (w,s) & \longmapsto & \begin{pmatrix} (\psi_1\chi)(w) & & \\ & S_3(s) & \\ & & (\psi_1\chi)(w)^{-1} \end{pmatrix} \end{array}$$

where S_3 is the irreducible representation of dimension 3 of $SL_2(\mathbb{C})$.

We have $\phi\left(1, \begin{pmatrix} \tau^{-1} & \\ & \tau \end{pmatrix}\right) = \begin{pmatrix} 1 & & & & \\ & \tau^{-2} & & & \\ & & 1 & & \\ & & & \tau^2 & \\ & & & & 1 \end{pmatrix}$

For the irreducible subquotient of $\psi_1\chi \times \psi_2\zeta \rtimes 1$ there are $(\rho(\psi_1\chi, \psi_2\zeta), 0)$.

For the irreducible subquotient of $\psi_1\chi \times \psi_2\zeta \rtimes 1$ there are $(\rho(\psi_1\chi, \psi_2\zeta), 0)$.
We find in this way the cocharacters and the decomposition.

$\mathfrak{s} = [T, \chi \boxtimes 1]$

point of $T_{\mathfrak{s}} /\!/ W_{\mathfrak{s}}$	cocharacter	Langands parameter	constituant	representation
$(e, (z_1, z_2))$		$(\psi_1\chi, \psi_2)_{\widehat{T}}$	$\psi_1\chi \times \psi_2 \ltimes 1$	$\psi_1\chi \times \psi_2 \ltimes 1$
$(e, (z, q^{-1}))$		$(\psi\chi, \nu)_{\widehat{T}}$	$\psi\chi \rtimes 1_{SL_2}$	$\psi\chi \times \nu \ltimes 1$
$(b, (z, 1))$		$\begin{pmatrix} (\psi_1\chi)(w) & & \\ & S_3(s) & \\ & & (\psi_1\chi)(w)^{-1} \end{pmatrix}$	$\psi\chi \rtimes St_{SL_2}$	
$(e, (z, -1))$		$(\psi\chi, \varepsilon)_{\widehat{T}}$	$\psi\chi \rtimes T_1^{\varepsilon}$	$\psi\chi \times \varepsilon \ltimes 1$
$(b, (z, -1))$		$(\psi\chi, \varepsilon)_{\widehat{T}}$	$\psi\chi \rtimes T_2^{\varepsilon}$	

$\mathfrak{s} = [T, \chi \boxtimes 1]$

point of $T_{\mathfrak{s}} /\!/ W_{\mathfrak{s}}$	cocharacter	Langands parameter	constituant	representation
$(e, (z_1, z_2))$	$(1,1)$	$(\psi_1\chi, \psi_2)_{\widehat{T}}$	$\psi_1\chi \times \psi_2 \ltimes 1$	$\psi_1\chi \times \psi_2 \ltimes 1$
$(e, (z, q^{-1}))$	$(1,1)$	$(\psi\chi, \nu)_{\widehat{T}}$	$\psi\chi \rtimes 1_{SL_2}$	$\psi\chi \times \nu \ltimes 1$
$(b, (z, 1))$		$\begin{pmatrix} (\psi_1\chi)(w) & & \\ & S_3(s) & \\ & & (\psi_1\chi)(w)^{-1} \end{pmatrix}$	$\psi\chi \rtimes St_{SL_2}$	
$(e, (z, -1))$	$(1,1)$	$(\psi\chi, \varepsilon)_{\widehat{T}}$	$\psi\chi \rtimes T_1^{\varepsilon}$	$\psi\chi \times \varepsilon \ltimes 1$
$(b, (z, -1))$		$(\psi\chi, \varepsilon)_{\widehat{T}}$	$\psi\chi \rtimes T_2^{\varepsilon}$	

$\mathfrak{s} = [T, \chi \boxtimes 1]$

point of $T_{\mathfrak{s}} /\!/ W_{\mathfrak{s}}$	cocharacter	Langands parameter	constituant	representation
$(e,(z_1,z_2))$	$(1,1)$	$(\psi_1\chi, \psi_2)_{\widehat{T}}$	$\psi_1\chi \times \psi_2 \ltimes 1$	$\psi_1\chi \times \psi_2 \ltimes 1$
$(e,(z,q^{-1}))$	$(1,1)$	$(\psi\chi, \nu)_{\widehat{T}}$	$\psi\chi \rtimes 1_{SL_2}$	$\psi\chi \times \nu \ltimes 1$
$(b,(z,1))$	$(1,\tau^{-2})$	$\begin{pmatrix} (\psi_1\chi)(w) & & \\ & S_3(s) & \\ & & (\psi_1\chi)(w)^{-1} \end{pmatrix}$	$\psi\chi \rtimes St_{SL_2}$	
$(e,(z,-1))$	$(1,1)$	$(\psi\chi, \varepsilon)_{\widehat{T}}$	$\psi\chi \rtimes T_1^{\varepsilon}$	$\psi\chi \times \varepsilon \ltimes 1$
$(b,(z,-1))$		$(\psi\chi, \varepsilon)_{\widehat{T}}$	$\psi\chi \rtimes T_2^{\varepsilon}$	

$$\mathfrak{s} = [T, \chi \boxtimes 1]$$

point of $T_{\mathfrak{s}} /\!/ W_{\mathfrak{s}}$	cocharacter	Langands parameter	constituant	representation
$(e,(z_1,z_2))$	$(1,1)$	$(\psi_1\chi, \psi_2)_{\widehat{T}}$	$\psi_1\chi \times \psi_2 \ltimes 1$	$\psi_1\chi \times \psi_2 \ltimes 1$
$(e,(z,q^{-1}))$	$(1,1)$	$(\psi\chi, \nu)_{\widehat{T}}$	$\psi\chi \rtimes 1_{SL_2}$	$\psi\chi \times \nu \ltimes 1$
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$(e,(z,-1))$	$(1,1)$	$(\psi\chi, \varepsilon)_{\widehat{T}}$	$\psi\chi \rtimes T_1^{\varepsilon}$	$\psi\chi \times \varepsilon \ltimes 1$
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$\mathfrak{t} = [T, \chi \boxtimes \zeta]$

point of $T_\mathfrak{t} /\!/ W_\mathfrak{t}$	cocharacter	Langands parameter	constituant	representation
$(e, (z_1, z_2))$	$(1, 1)$	$(\psi_1\chi, \psi_2)_{\widehat{T}}$	$\psi_1\chi \times \psi_2 \ltimes 1$	$\psi_1\chi \times \psi_2 \ltimes 1$
$(e, (z, 1))$	$(1, 1)$	$(\psi\chi, \zeta)_{\widehat{T}}$	$\psi\chi \rtimes T_1^{\zeta}$	$\psi\chi \times \zeta \ltimes 1$
$(b, (z, 1))$	$(1, 1)$	$(\psi\chi, \zeta)_{\widehat{T}}$	$\psi\chi \rtimes T_2^{\zeta}$	
$(e, (z, -1))$	$(1, 1)$	$(\psi\chi, \varepsilon\zeta)_{\widehat{T}}$	$\psi\chi \rtimes T_1^{\varepsilon\zeta}$	$\psi\chi \times \varepsilon\zeta \ltimes 1$
$(b, (z, -1))$	$(1, 1)$	$(\psi\chi, \varepsilon\zeta)_{\widehat{T}}$	$\psi\chi \rtimes T_2^{\varepsilon\zeta}$	

Thank you for your attention.

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