

# Abstracts

## Young Topologists Meeting 2012

*Hector Cordova Bulens*

### **Higher order linking numbers**

Let  $L: S^1 \cup S^1 \rightarrow S^3$  be a link. We can calculate the linking number of the two circles using a cup product in the cohomology of the complementary space  $S^3 \setminus L(S^1 \cup S^1)$ .

W.S. Massey introduced a second order cohomology operation, the triple product, which can be defined in terms of cup products of suitable cochains. We will show how to use this operation to detect non trivial links of three circles and in general of three higher dimensional spheres. As an example we will show how to differentiate the borromean link from the trivial one.

The talk is based on an article from W.S. Massey first published in 1968 and reedited in 1998.

*Rosona Eldred*

### **Calculus of Functors and Nilpotence**

A notion of nilpotence for spaces has been important in the use of localizations/completions, and often one can extend a previous result on simply connected spaces to those which are nilpotent. We will give an introduction to the calculus of functors, focusing on its role in providing a notion of "homotopy-correct" nilpotence for spaces and their classification, as well as related results. This will touch on recent work of Biedermann-Dwyer, Chorny-Scherer, as well as the speaker.

*Barbara Di Fabio*

### **Mayer-Vietoris formulas for persistent homology**

In algebraic topology, a Mayer-Vietoris formula represents a relationship among the ranks of homology groups of spaces  $X, A, B, C$  when  $X = A \cup B$  and  $C = A \cap B$ . It is a powerful tool for studying the homology of a space  $X$  by splitting it into subspaces whose homology is simpler to compute.

In [2, 3] it is shown that analogous formulas also hold for the ordinary, relative, and extended homology groups introduced in [1].

Persistent homology is an algebraic tool for shape analysis and description whenever a multidimensional data set can be modeled as a topological space  $X$ , and whose shape properties can be described by a continuous function  $f$  defined on it. Persistent homology groups are defined in terms of the lifetimes of homological classes along the filtration of  $X$  given by the lower level sets of  $f$ . The scale at which a feature is significant is measured by its persistence. The basic assumption is that the longer a feature survives, the more meaningful or coarse the feature is for shape description. Vice-versa, noise and shape details are characterized by a shorter life. Ordinary persistence takes into account only bounded lifetimes, while extended persistence the unbounded ones. Pairing births and deaths of topological features along the filtration generates a collection of points in the plane called a persistence diagram.

In this context, the validity of Mayer-Vietoris formulas yields relationships among the persistence diagrams of  $X$ ,  $A$ ,  $B$ . This indicates that the presence of  $A$  (respectively,  $B$ ) in  $X$  can be revealed by the presence of a subset of points of the persistence diagram of  $A$  (respectively,  $B$ ) in the persistence diagram of  $X$ . These results can be used to treat problems of partial similarity shape detection [4].

## References

- [1] D. Cohen-Steiner, H. Edelsbrunner, J. Harer: "Extending Persistence Using Poincaré and Lefschetz Duality", *Foundations of Computational Mathematics* 9 (2009), no. 1, 133-134.
- [2] B. Di Fabio, C. Landi: "A Mayer-Vietoris Formula for Persistent Homology with an application to Shape Recognition in the Presence of Occlusions", *Foundations of Computational Mathematics* 11 (2011), 449-527.
- [3] B. Di Fabio, C. Landi: "Persistent Homology and Partial Similarity of Shapes", *Pattern Recognition Letters* 33 (2012), 1445-1450.
- [4] B. Di Fabio, C. Landi, F. Medri: "Recognition of Occluded Shapes Using Size Functions", *Lecture Notes In Computer Science*, Vol 5716 (2009), 642-651.

*Fabian Hebestreit*

### **The Bordism Approach to Scalar Curvature**

In 1980 Gromov and Lawson, and independently Schoen and Yau, proved the existence problem for positive scalar curvature metrics on closed manifolds to be invariant under cobordism (provided dimensions are greater than four).

However the appropriate flavour of cobordism varies with the given manifold. I will sketch the application of this result in the easiest case, namely simply connected manifolds, try to give an idea of approaches to the

Gromov-Lawson-Rosenberg conjecture, which covers the case of spin manifolds, and present aspects of my own work addressing a similar conjecture for hardly spin manifolds, that is manifolds whose universal cover is spin.

*Robin Koytcheff*

### **A colored operad for infection of string links**

Ryan Budney constructed an operad that encodes splicing of knots and extends his little 2-cubes action on the space of (long) knots. He further showed that the space of knots is freely generated over the splicing operad by the subspace of torus and hyperbolic knots. Infection of knots (or links) by string links is a generalization of splicing from knots to links and is useful for studying concordance of knots. In joint work with John Burke, we construct a colored operad that encodes this infection operation.

*Alexander Kupers*

### **Higher string operations using radial slits configurations**

We describe an alternative method to construct higher string operations for manifolds. This method is based on Godin's construction, but replaces the geometric realisation of a category of decorated ribbon graphs by Bodigheimer's radial slit configurations model for the moduli space of Riemann surfaces with boundary. This significantly simplifies the construction and allows one to extend the construction a partial compactification of the moduli space. As a consequence one can easily derive relations between the higher string operations and define secondary operations.

*Cary Malkiewich*

### **Thom Spectra as Linear Approximations**

In 1999 M. Chas and D. Sullivan defined a new product on the homology of  $LM$ , the space of unbased loops in a closed oriented manifold  $M$ . In 2002 R.L. Cohen and J.D.S. Jones defined a ring spectrum  $LM^{\{-TM\}}$  giving the same product on the homology of  $LM$ . More generally, each bundle  $E \rightarrow M$  has an associated Thom spectrum  $E^{\{-TM\}}$ , which we can think of as a highly structured intermediary between  $E$  and the homology of  $E$ , and we can use this approach to understand products on  $E$  and its homology. We'll discuss a possible new approach to understanding this structure, using an unusual variant of Goodwillie calculus to construct a tower of polynomial approximations in which  $E^{\{-TM\}}$  is only the first level.

*Kristen Mazur*

### **A Characterization of Tambara Functors**

Tambara functors play a key role in equivariant homotopy theory. Indeed, the zeroeth stable homotopy groups of commutative  $G$ -ring spectra are Tambara functors. In this talk I will introduce Tambara functors, and give a description of the fixed point and Burnside Tambara functors. Further, for  $G$  a finite group and  $H$  a subgroup of  $G$ , I will define a norm map from  $H$ -Tambara functors to  $G$ -Tambara functors that is analogous to the norm map between  $H$ -spectra and  $G$ -spectra. Time permitting, I will use this map to give a new characterization of Tambara functors as commutative Mackey functor rings.

*Lennart Meier*

### **Module spectra over real K-theory and TMF**

K-theory and TMF are examples of ring spectra. We will study the categories of modules over them. These are related to the first two chromatic approximations to the stable homotopy category.

*Simon Naarmann*

### **Transfer in generalized cohomology theories**

We construct a natural transfer in cellular cohomology for ramified coverings. We then show that any generalized cohomology theory admitting such a transfer for finite-group orbit projections of

CW complexes must be ordinary.  
The proof is based on a sketch by A. Dold.

*Thomas Nikolaus*

### **Algebraic K-Theory of oo-Operads**

We first review the theory of dendroidal sets introduced by Moerdijk and Weiss '07. The notion of a dendroidal set is an extension of the notion of a simplicial set which has been introduced to serve as a combinatorial model for oo-operads. Moreover the category of dendroidal sets contains the category of symmetric monoidal categories as a full subcategory.

As a next step we define the algebraic K-Theory groups  $K_n(D)$  of a dendroidal set  $D$  generalizing the algebraic K-Theory groups of symmetric monoidal categories. We present some easy properties and calculations. The main result is that these groups are the homotopy groups of a spectrum  $KD$  and that the assignment  $D \rightarrow KD$  provides an equivalence between the homotopy category of connective spectra and a suitable localization of the homotopy category of dendroidal sets.

This construction solves the open problem of finding a geometric realization for dendroidal sets. It also provides a new proof that the algebraic K-Theory functor is compatible with monoidal structures, in particular that the algebraic K-Theory spectrum of a bipermutative category is a ring spectrum.

*Kazunori Noguchi*

### **The Euler characteristic and the zeta function for finite categories**

The Euler characteristic is defined for various mathematical objects, for example topological spaces, graphs and so on, but categories are also. Leinster defined the Euler characteristic for finite categories. It is generalization of the usual one for simplicial complexes and it has many interesting properties.

On the other hands, the zeta function is defined for finite categories.  
I'll talk about these two notions and their relationship.

*Sam Nolen*

### **Some applications of the cobordism hypothesis**

The Baez-Dolan cobordism hypothesis, as reformulated in terms of higher category theory by

Hopkins and Lurie, is a powerful result classifying topological field theories. It is known that the cobordism hypothesis implies the theorem of Galatius, Madsen, Tillmann, and Weiss on the homotopy type of the cobordism category. I will explain a bit about how this works, and describe a new application, a characterization of the homotopy type of the space of "infinity-gerbes" on a space.

*Martin Palmer*

### **Configuration spaces and homological stability**

Given a fixed background manifold  $M$  and parameter-space  $X$ , the associated configuration space is the space of  $n$ -point subsets of  $M$  with parameters drawn from  $X$  attached to each point of the subset, topologised in a natural way so that points cannot collide. One can either remember or forget the ordering of the  $n$  points in the configuration, so there are ordered and unordered versions of each configuration space. These spaces arise in many different areas of mathematics, so it is useful to understand for example their homology.

It is a classical result that the sequence of unordered configuration spaces, as  $n$  increases, is homologically stable: for each  $k$  the degree- $k$  homology is eventually independent of  $n$ . However, a simple counterexample shows that this is false for ordered configuration spaces. A natural question then is whether one can remember part of the ordering information and still retain homological stability.

The goal of this talk is to explain the ideas behind a positive answer to this question, using the notion of 'oriented configuration spaces', where configurations are equipped with an ordering of the points up to even permutations. I will also explain how this case differs from the unordered case: for example the 'rate' at which the homology stabilises is strictly slower for oriented configurations. Finally, if time permits, I will also say something about homological stability with twisted coefficients.

*Irakli Patchkoria*

### **Rigidity in equivariant stable homotopy theory**

Using the completion theorem (formerly known as the Segal conjecture), we prove that for any finite abelian 2-group  $G$ , the 2-local  $G$ -equivariant stable homotopy category, indexed on a complete  $G$ -universe, has a unique equivariant model in the sense of Quillen model categories. This means that the suspension functor, homotopy cofiber sequences and the stable Burnside category determine all "higher order structure" of the 2-local  $G$ -equivariant stable homotopy

category such as for example equivariant homotopy types of function  $G$ -spaces. The theorem can be seen as an equivariant generalization of Schwede's rigidity theorem at prime 2.

*Kate Poirier*

### **Compactifying string topology**

String topology studies the algebraic topology of the free loop space of a manifold. In this talk, we describe a compact space of graphs and show how this space gives algebraic operations on the singular chains of the free loop space. In particular, our chain level operations induce Cohen and Godin's "positive boundary TQFT" on the homology of the free loop space. This project is joint work with Nathaniel Rounds.

*Andrew Russhard*

### **$p^{\text{th}}$ power maps on quasi $p$ -regular Lie groups**

In the talk I propose to give an over view of a method I am attempting to use to show that the  $p^{\text{th}}$  power map on quasi- $p$ -regular  $SU(n)$  is an H-map. I will define what it means for a space to be quasi- $p$ -regular, and then present some results of Mimura & Toda, Kishimoto and Cohen, Moore & Neisendorfer (all without proof of course!). I will then go on to explain how I plan to use these results to show that the  $p^{\text{th}}$  power map on quasi- $p$ -regular  $SU(n)$  is an H-map. If time permits I will then discuss my results so far.

*Helene Sigloch*

### **Classification of Vector Bundles**

It is a well-known fact that vector bundles on a paracompact Hausdorff space  $X$  correspond to homotopy classes of maps into the Grassmannian and that every vector bundle on the product of  $X$  with the real line is the pullback of some vector bundle on  $X$ . Trying to understand a space by looking at its vector bundles is a concept that is not unique to Algebraic Topology. In this talk, we will look at the question how far analogous results hold in Algebraic Geometry.

*Mirjam Solberg*

### **Braided commutative monoids and double loop spaces**

In topological spaces, a double loop space is the space of based maps from the two-sphere into a

space with basepoint. If we identify all the points in the equator of a two-sphere we get a wedge of two two-spheres. We can use this to define a multiplication on a double loop space, making it a commutative monoid in the homotopy category.

Given a double loop space, we would like to find a commutative monoid that is weakly equivalent to the double loop space. This is not always possible. The second best thing is a category that is Quillen equivalent to topological spaces, in which commutative monoids correspond to double loop spaces.

This was done for infinite loop spaces by Sagave and Schlichtkrull recently, by constructing a symmetric monoidal diagram category where commutative monoids correspond to infinite loop spaces.

I will discuss a braided version of this construction, and explain why braided monoidal categories are the right place to look for double loop spaces.

*Matthias Spiegel*

### **K-Theory of Intersection Spaces**

My talk will deal with Poincare Duality in generalized homology theories for stratified pseudomanifolds. Poincare Duality for manifolds can be generalized to other homology theories. Stratified pseudomanifolds are spaces, that are not manifolds, but can be decomposed into subsets which are manifolds. For those, Poincare Duality does not hold in general, but this can be fixed using intersection homology. However, intersection homology involves truncation on the chain level and therefore cannot easily be applied to homology theories that do not factor through chain complexes. We will use a spatial truncation instead. This allows, to associate a topological space, called intersection space, to certain stratified pseudomanifolds, whose ordinary homology satisfies Poincare duality. We will then look at generalized homology theories, such as K-theory, of these intersection spaces.

*Kohei Tanaka*

### **Reconstruction of manifolds from their Morse functions**

I will introduce a work of Cohen, Jones and Segal about reconstruction of a manifold from a Morse function on itself, and I will give an alternative proof of it using a cell structure on the manifold. I would also like to talk about discrete Morse theory version of the above.



*Sebastian Thomas*

### **On the triangulated structure of homotopy categories**

As recently shown by Schwede, the homotopy category of a stable Brown cofibration category carries the structure of a triangulated category in the sense of Verdier. This generalises Hovey's theorem for stable Quillen model categories. So the homotopy category of a Brown cofibration category may be seen as equipped with an autofunctor and distinguished diagrams, called (Verdier) triangles, fulfilling a short list of axioms. One of them deals with so-called (Verdier) octahedra; it roughly states that triangles are compatible with composition. The usual proof of the octahedral axiom for homotopy categories yields in fact particular well-behaved octahedra, which fulfill properties similar to the axioms of a triangulated category. The goal of the talk is to indicate how this gives rise to a stronger structure on the stabilisation of the homotopy category of a Brown cofibration category, and how this structure can be generalised to the unstable case. The axioms in the stable case are due to Künzler and, independently, Maltziniotis.

*Neset Deniz Turgay*

### **Conjugation invariants in the Leibniz-Hopf algebra**

The Leibniz-Hopf algebra is the free associative algebra with one generator in each degree and co-product given by the Cartan formula. The mod 2 Steenrod algebra naturally occurs as a quotient of this Hopf algebra by the Adem relations.

The ring of conjugation invariants in the dual Steenrod algebra arises when one considers commutativity of ring spectra. Motivated by this, Martin Crossley and I have studied the fixed points in the mod 2 dual Leibniz-Hopf algebra under this conjugation action. We found that, like in the dual Steenrod algebra, these invariants are "approximately" half of the whole algebra, although we are able to give a much more precise statement than was possible for the Steenrod algebra.

In this talk, I will show how to describe the conjugation invariants in the (undualized) Leibniz-Hopf algebra, using the previous results in dual case.

*Markus Upmeyer*

### **Products in Differential Cohomology**

Differential cohomology groups merge local geometric data of a manifold, encoded as differential

forms, with global topological information.

Applications range from physics (charge quantization) to conformal geometry, where "refined invariants" are described with them.

In my talk, I plan to introduce differential cohomology along with its basic properties, report on my own work concerning multiplicative structures, and, if time permits, indicate an interpretation in terms of (higher) categories.

*Inna Zakharevich*

### **K-theory as a multifunctor**

The algebraic K-theory of a Waldhausen category "group completes" the coproduct in the category by producing a spectrum whose  $\pi_0$  is the Grothendieck group of the category. If we want to construct a Waldhausen category whose K-theory is an  $E_\infty$  ring spectrum (and, in particular, has a ring at  $\pi_0$ ) we need a biexact multiplication on the category. Generalizing this, we can define "multilinear" maps out of a product of Waldhausen categories with the "multiexact" functors and make the category of Waldhausen categories into a multicategory. In this talk we present a proof of the fact that the K-theory functor is a multifunctor and thus takes "multilinear" morphisms of Waldhausen categories to "multilinear" morphisms of spectra.

*Sebastian Öberg*

### **Continuity and classifications**

Classifications of fibrations, fiber bundles, exact sequences, extensions etc. have played a central role in algebraic topology and algebra. An important ingredient in these classifications is the notion of continuity of mapping spaces and spaces of weak equivalences. In my talk I will describe the concept of continuity in model categorical terms. I will present an explicit and simple model for mapping spaces in arbitrary model categories. This model turns out to be particularly useful to reprove classification results and construct universal objects. This will be illustrated by calculating the homotopy groups of the category of extensions, reproving the theorem of Retakh.

# Gong show titles

## Tuesday

*Tan Li:* **Combinatorial Novikov-Morse theory**

*Andrea Cesaro:* **Kan extension of homology theories**

*Torleif Veen:* **Periodic elements in K-theory**

*Ulrich Pennig:* **Higher twisted K-theory**

*Moritz Rodenhausen:* **Dehn twists of graphs of groups**

*María José Pereira-Sáez:* **On the Lusternik-Schnirelmann category of symmetric spaces**

## Thursday

*Peter Arndt:* **Motivic Homotopy Theory over deeper bases**

*Sam Nariman:* **Topology of conjugate varieties**

*Paul Arnaud Songhafou Tsopméné:* **Multiplicative Operads up to homotopy**

*Eric Malm:* **String Topology, Hochschild Constructions, and the Based Loop Space**

*Federico Cantero:* **Homological stability for spaces of surfaces**