Amplified Graph Algebras

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Joint work with
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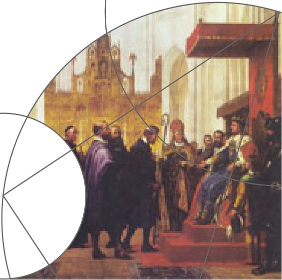

September, 2011

Graphs

Definition

A graph G is a 4-tuple (G^0, G^1, r, s), where G^0 is a countable set of vertices, G^1 is a countable set of edges, and $r, s\colon G^1 \to G^0$ are the range and source maps.

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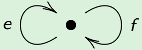

$$G^0 = \{\bullet\}$$

$$G^1 = \{e, f\}$$

$$s(e) = \bullet, r(e) = \bullet$$

$$s(f) = \bullet, r(f) = \bullet$$

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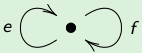

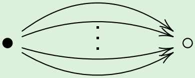

$G^0 = \{\bullet\}$	$G^0 = \{\bullet, \circ\}$
$G^1 = \{e, f\}$	$G^1 = \{e_1, e_2, \dots\}$
$s(e) = \bullet, r(e) = \bullet$	$s(e_1) = \bullet, r(e_1) = \circ$
$s(f) = \bullet, r(f) = \bullet$	$s(e_2) = \bullet, r(e_2) = \circ$

Graph C^*-Algebras

Definition

Given a graph G, we let $C^*(G)$ denote the universal C^*-algebra generated by pairwise orthogonal projections $\{p_u \mid u \in G^0\}$ and partial isometries $\{s_e \mid e \in G^1\}$ subject to the relations

CK0 $s_e^* s_f = 0$, if $e \neq f$.

CK1 $s_e^* s_e = p_{r(e)}$.

CK2 $s_e s_e^* \leq p_{s(e)}$.

CK3

$$p_u = \sum_{\{e \in G^1 \mid s(e) = u\}} s_e s_e^*,$$

if $0 < |s^{-1}(u)| < \infty$.

Examples of Graph C^*-Algebras

• The complex numbers.

The Cuntz algebra $\mathcal{O}_n$, $2 \leq n \leq \infty$.

The unitization of the compact operators.

The Kirchberg algebra with $K_0 = \mathbb{Z}^2$ and $K_1 = 0$.

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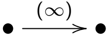

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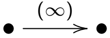

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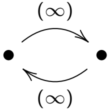

The Kirchberg algebra with $K_0 = \mathbb{Z}^2$ and $K_1 = 0$.

Classification and the Graphs

Question

Are graph algebras classified by K-theory?

Question

What does it say about two graphs E and G that $C^*(E)$ is (stably) isomorphic to $C^*(G)$?

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Is there a (finite) list of "moves" on graphs that generate the relation $G \sim E$ if and only if $C^*(G)$ is stably isomorphic to $C^*(E)$?

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Graph Operations

Definition

Given a graph G, we let the transitive closure of G be the graph $\mathtt{t}G$. It has the same vertex set as G and if there is a path from u to v in G, then there is an edge in $\mathtt{t}G$ with source u and range v.

Definition

Given a graph G, we define the amplification of G to be the graph $\overline{G}$ with the same vertex set as G, but with the property that if there is an edge from u to v in G, then there are infinitely many edges from u to v in $\overline{G}$.

Graph Operations - Pictures

G	$\mathtt{t}G$	$\overline{\mathtt{t}G}$

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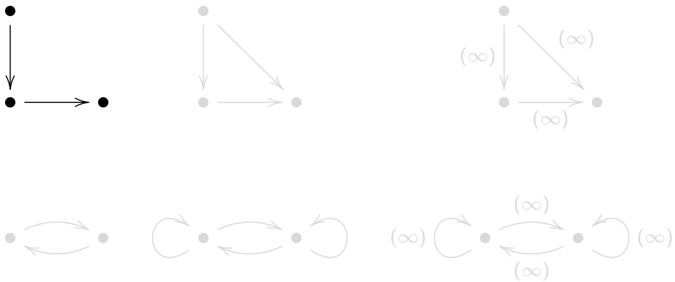

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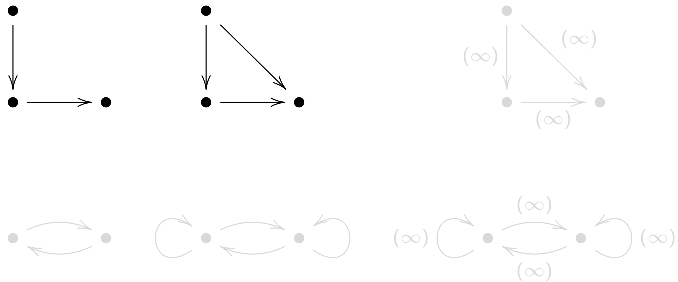

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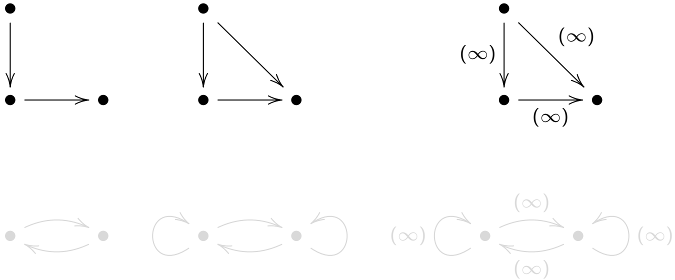

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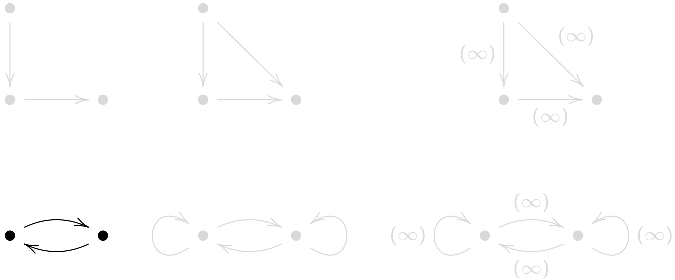

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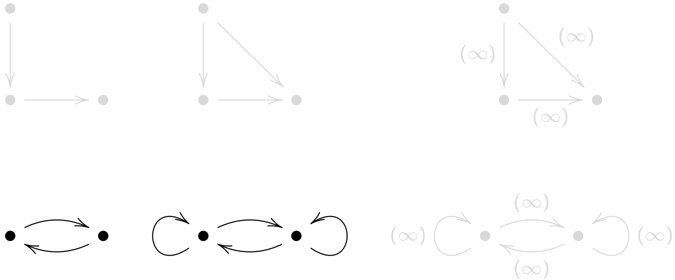

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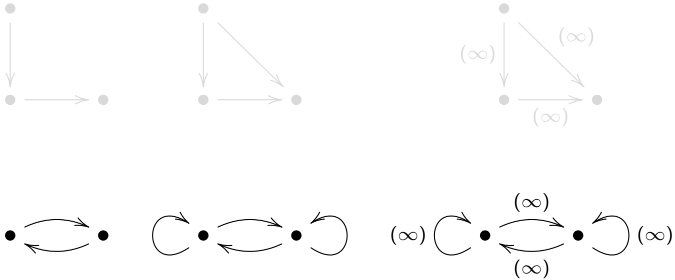

The Main Result

Theorem

Let G and E be finite graphs. The following are equivalent:

(i) $C^*(\overline{G}) \cong C^*(\overline{E})$.

(ii) $C^*(\overline{G})$ *and* $C^*(\overline{E})$ *have the same filtered* K*-theory.*

(iii) $C^*(\overline{\mathtt{t}G})$ *and* $C^*(\overline{\mathtt{t}E})$ *have the same filtered* K*-theory.*

(iv) $\overline{\mathtt{t}G} \cong \overline{\mathtt{t}E}$.

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Lemma A

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Lemma B

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Is It Interesting?

- Cons:
  - Specialized graphs.
  - Very boring K-theory ($K_0(C^*(\overline{G})) = \mathbb{Z}^{|G^0|}, K_1(C^*(\overline{G})) = 0$).
- Pros:
  - $C^*(\overline{G})$ can have any (finite) ideal structure.
  - Graphical classification.
  - Nice generalizations (graphs where all vertices are singular).
  - Permanence results (if it looks like an amplified graph algebra, and it quacks like an amplified graph algebra, then it must be an amplified graph algebra).

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Move (T)

Theorem

Let G be a graph, $u \in G^0$ an infinite emitter, and v a vertex that u emits infinitely to. Fix an edge $f \in s_G^{-1}(v)$. Let E be the graph with vertex set G^0, edge set

$$E^1 = G^1 \cup \{f^n \mid n \in \mathbb{N}\},$$

and range and source maps that extend those of G and have $r(f^n) = r(f)$ and $s(f^n) = u$. Then $C^*(E) \cong C^*(F)$.

Example

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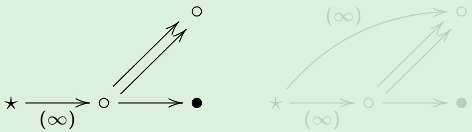

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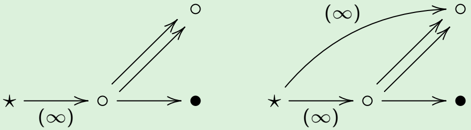

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Sketch of Proof

- For two vertices, u and v, write $u \geq v$ if there is a path from u to v or $u = v$.
- For amplified graphs, the relation $\geq$ is encoded in the ideal structure.
- Use $\mathrm{Prim}(C^*(\overline{\mathtt{t}G})) \cong \mathrm{Prim}(C^*(\overline{\mathtt{t}E}))$ to find a bijection $\psi\colon \overline{\mathtt{t}G}^0 \to \overline{\mathtt{t}E}^0$ such that $u \geq v \iff \psi(u) \geq \psi(v)$.
- We are done if we can show that a vertex supports a simple loop in $\overline{\mathtt{t}G}$ if and only if it does in $\overline{\mathtt{t}E}$.
- The ordered K_0-group is used to tell us this.

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Classification From the Outside

Definition

Let $\mathcal{C}$ be the class of separable, nuclear, simple, purely infinite C^*-algebras $\mathfrak{A}$ satisfying the UCT, and with $K_1(\mathfrak{A}) = 0$ and $K_0(\mathfrak{A})$ free.

Definition

Let $\mathcal{C}_{\mathrm{free}}$ be the class of C^*-algebras $\mathfrak{A}$ such that $\mathrm{Prim}(\mathfrak{A})$ is finite, and for every simple sub-quotient $\mathfrak{B}$ of $\mathfrak{A}$ we have

- $\mathfrak{B}$ is unital or stable, and in $\mathcal{C}$ or stably isomorphic to $\mathcal{K}$, and,
- if $\mathfrak{B}$ is unital, then there exists an isomorphism $K_0(\mathfrak{B}) \cong \bigoplus_n \mathbb{Z}$ such that $[1_{\mathfrak{B}}]$ is sent to $(1, \lambda)$.

Theorem

The elements of $\mathcal{C}_{\mathrm{free}}$ are classified by filtered K-theory.

Classification From the Outside

Definition

Let $\mathcal{C}$ be the class of separable, nuclear, simple, purely infinite C^*-algebras $\mathfrak{A}$ satisfying the UCT, and with $K_1(\mathfrak{A}) = 0$ and $K_0(\mathfrak{A})$ free.

Definition

Let $\mathcal{C}_{\mathrm{free}}$ be the class of C^*-algebras $\mathfrak{A}$ such that $\mathrm{Prim}(\mathfrak{A})$ is finite, and for every simple sub-quotient $\mathfrak{B}$ of $\mathfrak{A}$ we have

- $\mathfrak{B}$ is unital or stable, and in $\mathcal{C}$ or stably isomorphic to $\mathcal{K}$, and,
- if $\mathfrak{B}$ is unital, then there exists an isomorphism $K_0(\mathfrak{B}) \cong \bigoplus_n \mathbb{Z}$ such that $[1_{\mathfrak{B}}]$ is sent to $(1, \lambda)$.

Theorem

The elements of $\mathcal{C}_{\mathrm{free}}$ are classified by filtered K-theory.

The Payoff

Proposition

Let G be a graph with finitely many vertices, no breaking vertices, and with the property that every vertex in G is either an infinite emitter or a sink. We have $C^*(G) \in \mathcal{C}_{\text{free}}$.

Theorem

Let $\mathfrak{A}$ be a unital C^*-algebra in $\mathcal{C}_{\text{free}}$ with $K_0(\mathfrak{A})$ finitely generated. There exists a finite graph G such that $\mathfrak{A} \cong C^*(\overline{G})$.

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Let G_1 and G_2 be finite graphs. If $\mathfrak{A}$ is a unital C^*-algebra and $\mathfrak{A}$ fits into the following exact sequence

$$0 \to C^*(\overline{G_1}) \otimes \mathcal{K} \to \mathfrak{A} \to C^*(\overline{G_2}) \to 0$$

then $\mathfrak{A} \in \mathfrak{C}_{\text{free}}$. Consequently, $\mathfrak{A} \cong C^*(\overline{G})$ for some finite graph G.

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This Is the End