ABSTRACTS
"SPECTRAL THEORY AND PARTIAL DIFFERENTIAL EQUATIONS"
UNIVERSITY OF COPENHAGEN
19 - 20 - 21 NOVEMBER 2008

IN HONOUR OF PROFESSOR GERD GRUBB

1. **Helmut Abels:**
On Stokes operators with variable viscosities

We will present some recent results on properties of some modified Stokes operators for the case that the viscosity is a given non-constant function. It will be shown that the Stokes operators admit a bounded H^∞-calculus in a certain class of bounded and unbounded domains. In particular, this implies maximal regularity of the Stokes operators and implies a characterization of the domains of the fractional powers of the Stokes operators. The proof is based on a parametrix construction with the aid of the calculus of pseudodifferential boundary value problems with non-smooth symbols.

2. **Jean-Michel Bony:**
Generalized Fourier integral operators and evolution equations

A classical Fourier integral operator is associated to a canonical transformation of the phase space which is homogeneous of degree one in the frequency variable. Two properties are important : 1. the conjugate of a classical pseudodifferential operator by such a FIO is itself a classical PsDO, the principal symbol being transported by the canonical transformation. 2. the propagators of a strictly hyperbolic equation are FIO whose canonical transformation is given by the flow of the hamiltonian vector field associated to the equation.

This talk will develop an analogous theory for more general evolution equations, including Schrödinger ones. The canonical transformation is no longer homogeneous and property 1 should be replaced by the fact that conjugates of operators belonging to some class of generalized PsDO (i.e., in the Weyl-Hörmander calculus, associated to a Riemannian metric on the phase space) belong to another class of PsDO (i.e. associated to another metric). There is a symbolic calculus for these generalized FIO, where the group of metaplectic operators plays a crucial role.

3. **Louis Boutet de Monvel:**
Asymptotic equivariant index of Toeplitz operators and Atiyah-Weinstein conjecture

The equivariant index of transversally elliptic systems (for a compact group action) was introduced and studied by Atiyah. Here we introduce an avatar: the asymptotic index (the singularity of Atiyah's index, which is a distribution on G). This still makes sense for equivariant Toeplitz operators on contact manifolds. This was used in a joint work with E. Leichtnam, X. Tang, A. Weinstein, to give a new natural proof of the Atiyah–Weinstein conjecture.

4. **Jochen Brüning:**
Fredholm analysis with singularities of conic type

This is a report on joint work with Bob Seeley. We consider a Riemannian manifold with a singularity which is formed by a bundle of metric cones. We consider further the Dirac operator on differential forms and construct a selfadjoint extension for this operator which admits a good parametric of pseudodifferential type. We discuss Fredholm properties and index formulae, and we propose a generalization to more general singularities.

5. **Nils Dencker:**
Solvability and Subellipticity for Systems of Differential Operators

For scalar pseudodifferential operators of principal type, solvability is equivalent to condition (Ψ) by the recent resolution of the Nirenberg-Treves conjecture. This condition involves only the sign changes of the imaginary part of the principal symbol along the bicharacteristics of the real part. Subellipticity for an operator is equivalent to (Ψ) together with the Hörmander bracket condition on the principal symbol.

For square systems there are no corresponding results. In fact, condition (Ψ) on the eigenvalues of the principal symbol is neither necessary nor sufficient for solvability, except in the case of constant characteristics. Also, the conditions for subellipticity are not known except in the diagonalizable case.

In this talk, we shall present some examples and new results on the solvability and subellipticity for square systems of principal type. These are the systems whose principal symbol vanishes of first order on its kernel.

6. **Daisuke Fujiwara:**
The second term of the semi-classical asymptotic expansion for Feynman path integrals with integrand of polynomial growth

I shall explain followings: 1) The time slicing approximation method for Feynman path integrals with a smooth potential leads us to a kind of oscillatory integrals over a space of large dimension. 2) A rather precise remainder estimate, which does not depend on dimensionality, of stationary phase method for such oscillatory integrals is useful to give rigorous meaning to Feynman path integrals. 3) Moreover, it gives an analytic expression for the second term of the semiclassical asymptotic expansion of Feynman path integrals.

7. **Giuseppe Geymonat:** CONTROLLABILITY OF THIN LINEARLY ELASTIC SHELLS

Thanks to the pioneering works of J.L. Lions there exists a general tool for the study of the the exact controllability of a distributed system, in particular of various shell models. More precisely let us suppose that it is possible to act on (at least) a part of the boundary of a thin, linearly elastic and isotropic shell with suitable boundary conditions. Then null (or exact) controllability consists in proving that starting from an arbitrary initial state it is possible to steer the shell to rest, by a proper choice of the boundary control, in a finite time. In the Koiter model the membrane and flexural deformation energy are defined by the symmetric forms

$$a_M(\mathbf{u},\mathbf{v}) = \int_\omega a^{\alpha\beta\lambda\mu}\gamma_{\alpha\beta}(\mathbf{u})\gamma_{\lambda\mu}(\mathbf{v})\sqrt{a}dy \tag{1}$$

$$a_F(\mathbf{u},\mathbf{v}) = \int_\omega a^{\alpha\beta\lambda\mu}\rho_{\alpha\beta}(\mathbf{u})\rho_{\lambda\mu}(\mathbf{v})\sqrt{a}dy \tag{2}$$

where $\gamma_{\alpha\beta}(\mathbf{u})$ and $\rho_{\alpha\beta}(\mathbf{u})$ are the linearized change of metric and of curvature tensors associated to $\mathbf{u}$. Let $\epsilon \geq 0$ a real parameter and let be $\mathbf{A}_\epsilon = \mathbf{A}_M + \frac{\epsilon^2}{3}\mathbf{A}_F$ the operator in $\mathbf{H} = L^2(\omega)^3$ associated to the bilinear form $a_\epsilon(\mathbf{u},\mathbf{v}) = a_M(\mathbf{u},\mathbf{v}) + \frac{\epsilon^2}{3}a_F(\mathbf{u},\mathbf{v})$ defined on a suitable subspace $\mathbf{V}$ of kinematically admissible displacements. Let us consider the evolution problem:

$$\begin{cases} \frac{\partial \mathbf{u}}{\partial t^2}(y,t) + \mathbf{A}_\epsilon \mathbf{u}(y,t) = 0, \quad \text{for} \quad y \in \omega \quad \text{and} \quad t > 0 \\ \mathbf{u}(y,0) = \mathbf{u}^0(y), \frac{\partial \mathbf{u}}{\partial t}(y,0) = \mathbf{u}^1(y) \quad \text{for} \quad y \in \omega \\ \mathbf{B}\mathbf{u}(y,t) = \mathbf{v}(y,t) \qquad \text{for} \quad y \in \gamma \quad \text{and} \quad t \geq 0 \end{cases} \tag{3}$$

where $\mathbf{B}$ is a suitable system of boundary conditions. The system is *exactly controllable in time* T if given an initial data $(\mathbf{u}^0, \mathbf{u}^1)$ it is possible to find a control $\mathbf{v}$ that can drive the system (3) to rest at time T i.e.

$$\mathbf{u}(y,T) = \mathbf{0}, \frac{\partial \mathbf{u}}{\partial t}(y,T) = \mathbf{0} \quad \text{for} \quad y \in \omega$$

In [1] it is for instance proved the exact controllability when the middle surface of the shell satisfies a suitable condition (e.g. is not too far from a plane). It is natural to study the dependence of the controllability time T on ϵ. Let us remark that when $\epsilon \to 0$ one has a singular perturbation problem; moreover the operator $\mathbf{A}_0 = \mathbf{A}_M$ has an essential

spectrum. One can then prove [2] a non-controllability result: there exist some in initial data $(\mathbf{u}^0, \mathbf{u}^1)$ such that the evolution system (3) is not exactly controllable.
It seems natural to address the following problem:

In the case $\epsilon = 0$ find a space of controllable initial data $(\mathbf{u}^0, \mathbf{u}^1)$.

I will give some preliminary partial results obtained in a joint work with Farid Ammar-Khodja and Arnaud Münch. The characterization of the controllable initial data depends on the study of the spectral problem associated to the operator $\mathbf{A}_M$. This will be illustrated on the example of hemispherical shells and on the example of an arch.

References

[1] Miara B., Valente V., *Exact controllability of a Koiter Shell by a Boundary Action*, J. Elasticity **52** 267-287 (1999).

[2] Geymonat G., Valente V., *A noncontrollability result for systems of mixed order*, SIAM J. Control Optim , **39 (3)**, 661-672 (2000).

8. **Peter Gilkey:**

Heat content and heat trace asymptotics with singular weighting functions

Let M be a compact Riemannian manifold with smooth non-empty boundary. Let r be the geodesic distance to the boundary. Let ϕ be a function which is smooth on the interior of M so that $r^\alpha \phi$ is smooth near the boundary for $\alpha > 0$. (In other words, ϕ is allowed to 'blow up' at a controlled rate near the boundary). We study the heat content asymptotics with initial temperature ϕ and the heat trace asymptotics $Tr(\phi e^{-t\Delta})$. This extends classical results from the smooth setting to a singular context. This is joint work with M. van den Berg, K. Kirsten, and R. Seeley.

9. **Bernard Helffer:**

Minimal spectral partitions and Aharonov-Bohm Hamiltonians

Given an open set Ω and a partition of Ω by k open sets ω_j, we can consider the quantity $\max_j \lambda(\omega_j)$ where $\lambda(\omega_j)$ is the groundstate energy of the Dirichlet realization of the Laplacian in ω_j. If we denote by $\mathfrak{L}_k(\Omega)$ the infimum over all the k-partitions of $\max_j \lambda(\omega_j)$, a minimal spectral k-partition is then a partition which realizes the infimum. Although the analysis is rather standard when $k = 2$ (we find the nodal domains of a second eigenfunction), the analysis of higher k's becomes non trivial and quite interesting. In this talk, we would like to discuss the properties of minimal spectral partitions, illustrate the difficulties by considering simple cases like the disc, the square ($k = 3$) or the sphere and will also exhibit the possible role of the hexagone in the

asymptotic behavior as $k \to +\infty$ of $\mathfrak{L}_k(\Omega)$.
This work has started in collaboration with T. Hoffmann-Ostenhof and has been continued (published or in progress) with as other coauthors S. Terracini, G. Vial and V. Bonnaillie-Noël.

10. **Rafe Mazzeo:**
A HEAT EQUATION ANOMALY

I will discuss an anomaly that arises when comparing the heat invariants of a polyhedral domain with the limits of the heat invariants of a smoothing of this domain. This defect turns out to have a simple explanation in terms of certain renormalized heat invariants of an associated complete manifold.

11. **Anders Melin:**
BACKSCATTERING TRANSFORMS AND MULTILINEAR HARMONIC ANALYSIS

An analysis of the antidiagonal part of the scattering matrix in quantum mechanics motivates the introduction of a backscattering transformation $X \ni v \mapsto B(v) \in Y$, where X is some suitable weighted Sobolev space in $\mathcal{D}'(\mathbf{R}^n)$ (with $n \geq 3$ odd) and Y is some subspace of $\mathcal{D}'(X)$. This transformation is a natural generalization of the 'scattering transformation' that appears in the Gelfand-Levitan-Marchenko theory and which has the remarkable property of linearizing the Korteweg-de Vries equation. Due to the high nonlinearity of B and its special scaling properties, it is difficult to find appropriate spaces X and Y between which B acts continuously. In the case $n = 3$ it was proved by R. Lagergren that B is an analytic mapping in a neighbourhood of the origin in X if $X = \{v \in L^1(\mathbf{R}^3); \nabla v \in L^1\}$.

In my talk I shall present some results about B when X is an L^2 Sobolev space. It turns out that B is entire analytic from $X \cap \mathcal{E}'$ to Y when $X = H_{(s)}(\mathbf{R}^n)$, $s \geq m = (n-3)/2$ and $Y = H^{\mathrm{loc}}_{(s)}(\mathbf{R}^n)$. The $B_k(v)$ appearing in the power series expansion $B(v) = \sum_1^\infty B_k(v)$ correspond to k-linear mappings B_k from X^k to $\mathcal{D}'$ which preserve C_0^∞. These operators have many interesting properties, and an analysis of them uses technique that reminds of some used in pseudo-differential calculus. The case when $k = 2$ is of special interest. It is shown that $B_2(v_1, v_2)$ inherits regularity as well as decay from each of the factors v_j. If for example $0 \leq \bar{a} \leq a, 0 \leq \bar{b} \leq b, \bar{a} + \bar{b} < 1/2$ then B_2 is continuous from $H_{(1/2+a,m+b)} \times H_{(1/2+a,m+b)}$ to $H_{(1/2+a+\bar{a},m+b+\bar{b})}$, where $H_{(a,b)} = \{u, (1+|x|^2)^{a/2}(1+|D|^2)^{b/2}u \in L^2\}$. There are similar results concerning local regularity for $B_k(v_1, \dots, v_k)$ when $k \geq 3$.

In my lecture I shall also illustrate some phenomena by discussing $B_2(v)$ when v is rotation invariant and $n = 3$. Then $B_2(v)$ has a simple analytical expression, and there are some unexpected causality relations showing that the restriction of $B_2(v)$ to any ball centered at the origin depends only on the restriction of v to the same ball.

All results that I am going to present are obtained in colloboration with Ingrid Beltiţă at the Intitute of Mathematics 'Simion Stoilow' of the Roumanian Academy in Bucharest.

12. **Richard Melrose:** Transmission and zero pseudodifferential operators

I will discuss the relationship between the transmission condition for pseudodifferential operators at a hypersurface, which is the basis of the algebra introduced by Boutet de Monvel, and the zero calculus introduced by Mazzeo in his thesis.

13. **Elmar Schrohe:** Bounded H_∞-calculus for Pseudodifferential Operators

In the theory of parabolic pde, maximal regularity has proven a very efficient tool to establish existence of short time solutions to nonlinear equations through a careful study of the linearized problem. Maximal regularity for the linear problem on the other hand is easily established by showing the existence of a bounded H_∞ calculus. In this talk I will try to show how such a calculus can be constructed fairly easily for large classes of pseudodifferential operators.

14. **Bert-Wolfgang Schulze:** The anti-transmission property

The pseudo-differential calculus of boundary value problems, contains the parametrices of (Shapiro-Lopatinskij) elliptic boundary value problems for differential operators and yields regularity of solutions with Sobolev smoothness up to the boundary. The latter comes from the respective mapping behaviour of pseudo-differential operators with the transmission property with a corresponding control up to the boundary. These properties are studied in the works of Boutet de Monvel [1], Grubb [3], [4], and other authors, cf. [2], [5].

The transmission property on the level of (say, zero order) homogeneous principal symbols implies a symmetry normal to the boundary. Most of the classical pseudo-differential symbols that are smooth up to the boundary do not share this property. For instance, symbols connected with the Zaremba problem, reduced to the boundary, are (after a reduction to order zero) antisymmetric and thus have what we call the anti-transmission property (at the respective interface on the boundary).This is a special case of other variants of violated transmission property, occurring in parametrices of mixed and other singular problems, cf. [5], [6] . From the point of view of the violated transmission property the calculus of boundary value problems is of analogous structure as that of pseudo-differential operators on a manifold with smooth edges. The substitute of the boundary symbols are edge symbols operating in weighted spaces on an infinite cone transversal to the edge. Also many other structures in boundary value problems have analogues in edge problems and vice versa. Edge degeneration or violated transmission property cause a subtle mapping behaviour. For instance, smoothness up to the boundary is mapped to more general asymptotics with complex powers of the normal variable, including logarithmic terms, cf. [7].

References

[1] L. Boutet de Monvel, *Boundary problems for pseudo-differential operators,* Acta Math. **126** (1971),11-51.
[2] G. I. Eskin, *Boundary value problems for elliptic pseudodifferential equations,* Math. Monographs, vol. 52, Amer.Math.Soc., Providence, Rhode Island, 1980, Transl. of Nauka, Moskva, 1973.
[3] G. Grubb, *Functional calculus of pseudo-differential boundary problems. Second Edition,* Birkhäuser Verlag, Boston, 1996.
[4] G. Grubb, *Singular Green operators and their spectral asymptotics,* Duke Math. J. **51** (1984), 477-528.
[5] S. Rempel and B.-W. Schulze, *Parametrices and boundary symbolic calculus for elliptic boundary problems without transmission property,* Math. Nachr. **105** (1982), 45-149.
[6] B.-W. Schulze, *Pseudo-differential boundary value problems, conical singularities, and asymptotics,* Akademie Verlag, Berlin, 1994.
[7] A. Volpato and B.-W. Schulze, *Operators with variable branching asymptotics on manifolds with edges,* (in preparation).

15. **Simon Scott:**
Determinant Structures

A determinant functional on an algebra/semigroup/category is a combination of more fundamental structures which are of some independent interest. For example, the even and odd Chern characters in algebraic and topological K-theory are cases of such structures, as indeed is the Fredholm index and also Melrose's suspended eta invariant. On the other hand, there are exotic determinants and quasi-determinants which in the case of psdos on closed manifolds are pretty well understood, but more generally are yet to be properly analyzed. I will talk about various instances of these structures, and in particular take a look at the case of the bordism category.

16. **Johannes Sjöstrand:**
Weyl asymptotics for non-self-adjoint differential operators with small random perturbations

Due to spectral instability (pseudospectrum) the eigenvalues of non-self-adjoint differential operators are often highly unstable under small perturbations. There are now several results stating that when we add a small random perturbation, we get Weyl asymptotic distribution of eigenvalues, with probability close to 1 in the semi-classical limit, and almost surely in the limit of large eigenvalues. We describe some of these results, due to M. Hager, W. Bordeaux-Montrieux and the speaker, as well some underlying ideas and proofs.

17. **Andre Unterberger:**

Pseudodifferential analysis for export to harmonic analysis and number theory

There is an analysis on the real line, of a violently non-classical type, in which the spectrum of the harmonic oscillator is the set of all integers. Nevertheless, the whole part of classical analysis that centers around the Heisenberg representation, the Fourier transformation or more generally the metaplectic representation, and the Weyl calculus, has counterparts in alternative analysis: only, the (positive-definite) scalar product of two functions must be replaced by an indefinite pseudoscalar product. The existence of the alternative pseudodifferential analysis makes it possible to put the (very popular at present) Rankin-Cohen brackets of holomorphic modular forms on the same footing as their non-holomorphic analogues (largely ignored from number-theorists) which arise in a natural way from the sharp composition of automorphic symbols in the Weyl calculus. Even though possible applications to P.D.E.'s are at present questionable, there are some very special problems in quantum mechanics which seem to call for the use of generalizations of the new analysis.