Optimal stopping

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My main area of research is stochastic calculus and its applications to e.g. stochastic control, sequential testing, quickest detection, and filtering. A large proportion of my research has been devoted to the theory and applications of optimal stopping.

Optimal stopping problems originated in Wald's sequential analysis (1947). In a general setting, Snell (1952) characterized the solution by means of the smallest supermartingale dominating the gain process (these methods are often referred to as martingale methods). In a Markovian setting, Dynkin (1963) characterized the solution by means of the smallest superharmonic function dominating the gain function (these methods are often referred to as Markovian methods). In the latter context there is a one-to-one correspondence between optimal stopping problems (in probability) and free-boundary problems (in analysis). Some of the applications of optimal stopping are in fields of mathematical statics (e.g. sequential analysis), stochastic analysis (e.g. sharp inequalities), and mathematical finance/engineering (e.g. arbitrage free prices of American put options, one of the best-known specific problems of optimal stopping).

An example of my research is the following. Imagine an investor who owns a stock which he wishes to sell so as to maximise his return. In line with the mean-variance analysis of Markowitz, the problem is to identify the return with the expectation of the terminal wealth and the risk with the variance of the terminal wealth. The quadratic nonlinearity of the variance then moves the resulting optimal stopping problem outside the scope of the standard optimal stopping theory which may be viewed as linear programming in the sense of optimising linear functions subject to linear constraints. Consequently the results and methods of the standard/linear optimal stopping theory are not directly applicable in this nonlinear setting. The investor aims to maximise the expectation of X_τ over all stopping times τ of X such that the variance of X_τ is bounded above by a positive constant. Similarly the investor could aim to minimise the variance of X_τ over all stopping times τ of X such that the expectation of X_τ is bounded below by a positive constant. A first application of Lagrange multipliers implies that the Lagrange function (Lagrangian) for either/both constrained problems can be expressed as a linear combination of the expectation of X_τ and the variance of X_τ with opposite signs. Optimisation of the Lagrangian over all stopping times τ of X thus yields the central optimal stopping problem. Due to the quadratic nonlinearity of the variance, standard results of the optimal stopping theory can no longer be applied. Conditioning on the size of the expectation, a second application of Lagrange multipliers reduces the nonlinear optimal stopping problem to a family of linear optimal stopping problems. The optimal stopping time depends on the initial point of X in an essential way. This spatial inconsistency introduces a time inconsistency in the problem, raised the definition of dynamic optimality in which each new position of the process X yields a new optimal stopping problem to be solved upon overruling all the past problems. This in effect corresponds to solving infinitely many optimal stopping problems dynamically in time with the aim of determining when it is optimal to stop in the sense that no other stopping time could produce a more favourable value in the future. Remarkably the solution of the dynamic formulation satisfies the principle of smooth fit which is known to be a key variational principle for linear optimal stopping problems under natural conditions.

The methodology of dynamic optimality was done in parallel for solving nonlinear stochastic control problems for tackling an optimal mean-variance portfolio selection problem. This topic is related to recent joint work with Goran Peskir (Manchester), Peter Johnson (KU), and Yumo Zhang (KU).

In 2007, there was a television programme on DR2: Viden Om: "Vind oftere - stop i tide" devoted to optimal stopping (here is link to the programme

matematikweb.dk/video%20fra%20tv/viden%20om%20vind%20oftere%20stop%20i%20tide.mp4

Mogens Steffensen from the department participated in the programme.