

CORNELIA DRUTU

## **$C^*$ -algebras and Geometric Group Theory**

In this mini-course I will overview various topics at the interface between  $C^*$ -algebras and Geometric Group Theory, from the Rapid Decay property to Kazhdan projections and various versions of amenability.

ILIJAS FARAH

## **Massive $C^*$ -algebras and their Applications**

Given a separable  $C^*$ -algebra  $A$  one can define the asymptotic sequence algebra  $\ell_\infty(A)/c_0(A)$ . By taking further quotients, one obtains various other extensions of  $A$ , the most important of which are the ultrapowers. When  $A$  is separable, the relation between  $A$  and these ‘massive’  $C^*$ -algebras is used to provide information about  $A$  itself. Quite surprisingly, this approach is frequently more efficient than the direct study of  $A$  itself. In these lectures, I will provide a friendly introduction to asymptotic  $C^*$ -algebras and ultrapowers, and show that in a certain precise sense the latter construction is, for all practical purposes, more general (i.e., more useful).

NADIA S. LARSEN

## **Equilibrium states on Nica-Toeplitz $C^*$ -algebras**

Equilibrium states have their origin in statistical mechanics. Their operator algebraic manifestation has provided a successful vein of investigation of many  $C^*$ -algebras, fostering rich interaction with fields such as number theory and ergodic theory. Equilibrium states for one-parameter groups on Toeplitz and Cuntz-Pimsner  $C^*$ -algebras associated to a  $C^*$ -correspondence were studied in work of Pinzari, Watatani and Yonetani, and of Laca and Neshveyev, almost two decades ago. These results have found numerous counterparts in modern examples of Toeplitz and Cuntz-Pimsner type  $C^*$ -algebras associated to semigroups of  $C^*$ -correspondences based on Nica’s quasi-lattice ordered groups. In this talk I will present an overview of this development and discuss recent findings from joint work with Z. Afsar and S. Neshveyev.

IAN F. PUTNAM

## **$C^*$ -algebras and Dynamics**

The main goal of the lectures is to provide an introduction to the connections between topological dynamical systems and  $C^*$ -algebras. This interaction has been fruitful for both fields, going back to the seminal work of Murray and von Neumann. On the one hand,  $C^*$ -algebras constructed from dynamical systems provide important examples. On the other,

$C^*$ -algebraic techniques and tools, particularly  $K$ -theory, provide interesting new insights into the dynamical systems.

The lectures will begin with the construction of  $C^*$ -algebras from groupoids, since this is at the heart of the interaction. We will briefly discuss  $K$ -theory, particularly for groupoid  $C^*$ -algebras and crossed products. We will also give a substantial list of topological dynamical systems, simply to provide a first insight into the types of behaviors that are possible, discussing the  $C^*$ -algebras briefly in each case. These include minimal systems,  $AF$ -equivalence relations, Smale spaces and examples from the field of aperiodic tilings.

Finally, we discuss applications of relative  $K$ -theory to the dynamics. The first example of such results was in the so-called orbit-splitting subalgebras of crossed products, which have played a key part in Elliott's classification program, at least as it applies to crossed products. We will give some more recent results along these lines, with applications to such systems as the dynamics of flat surfaces and some problems in the Elliott classification scheme.