2020.11.13

Complex Analysis

Recall from Peter's last lecture that for all $0 < p \le 1$, there is a full abelian subcat.

$$\mathrm{Liq}_p(\mathbb{R}) \subset \mathrm{Cond}(\mathbb{R})$$

of p-liquid cond. $\mathbb{R}$-v.s., which satisfies the axioms (*) of Dustin's lecture. So $\mathrm{Liq}_p(\mathbb{R})$ is closed under all colimits and limits and admits a symmetric monoidal p-liquid tensor product $- \otimes^{\mathrm{Liq}_p} -$ that preserves colimits in each factor. The ab. cat. $\mathrm{Liq}_p(\mathbb{R})$ is compactly generated, with generators given by the spaces of measures

$$\mathcal{M}_{<p}(S)$$

for S extremally disconnected profinite. This was obtained by replacing the 1-summability for Radon measures by p'-summability for all $p' < p$. The $\mathbb{R}$-v.s. $\mathcal{M}_{<p}(S)(*)$ consists only of

countable sums of Dirac measures, so this begs the question:

Are liquid modules complete enough?

Test: Calculate some liquid tensor products.

Note that for all $0 < p \leq 1$,

$$\mathcal{M}\text{-complete} \subset \mathrm{Liq}_p(\mathbb{R}).$$

<u>Ex</u> If V, W are real Banach spaces, then

$$V \otimes^{\mathrm{Liq}_p} W$$

is <u>not</u> a Banach space. In fact, it is not even locally convex. It is less complete that the least complete of the tensor products of Banach spaces, namely, the projective tensor prod. $V \otimes^{\pi} W$. Same if V, W are Smith sp. (For V, W Banach, also no reason to expect that the derived liquid tensor prod. is conc. in degree zero.)

So this seems discouraging.

By definition, for S, T profinite (and not nec. extr. disconn.),

$$\mathcal{M}_{<p}(S) \otimes^{Liqp} \mathcal{M}_{<p}(T)$$
$$\simeq \mathcal{M}_{<p}(S \times T).$$

Special case, for $S = \mathbb{N} \cup \{\infty\}$, $\mathcal{M}_{<p}(S)$ contains $\mathbb{R}$ as a summand by functoriality,

$$\mathbb{R} \longrightarrow \mathcal{M}_{<p}(S) \longrightarrow \mathbb{R}.$$

$$\{\infty\} \hookrightarrow \mathbb{N} \cup \{\infty\} \longrightarrow *$$

The complementary summand,

$$\mathcal{M}_{<p}(S) \simeq \mathbb{R} \oplus \ell^{<p}_{\mathrm{Smith}},$$

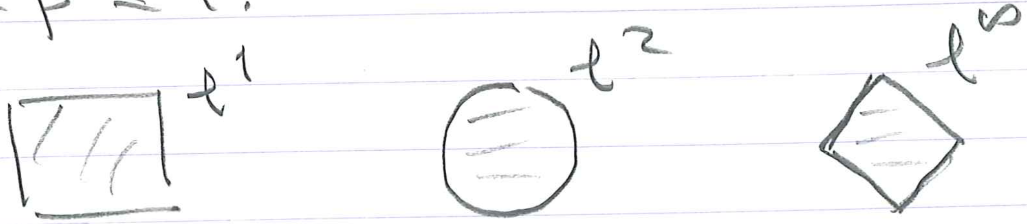
is the $<p$-Smith space of the usual sequence space, i.e.

$$\ell^{<p}_{\mathrm{Smith}} = \left\{ (\lambda_n)_{n \in \mathbb{N}} \;\middle|\; \begin{array}{l} \sum |\lambda_n|^{p'} < \infty \\ \text{for some } p' < p \end{array} \right\}.$$

It is the "free p-liquid $\mathbb{R}$-v.s. on a null-convergent seq."

Recall idea of Grothendieck in functional analysis.

In classical functional analysis have $1 \leq p < \infty$ in addition to $0 < p \leq 1$.



This distinction does not appear in p-adic functional analysis, due to ultrametric property of p-adic metric.

Grothendieck nuclearity, which similarly erases this distinction. Recall that there is a notion of "trace-class" or "1-summable" maps of Banach spaces $V \to W$.

<u>Ex</u> Basic example given by

$$\ell^\infty \xrightarrow{(\lambda_n)} \ell^1$$

with $\sum |\lambda_n| < \infty$. Any trace-class map factors through one of these,

$$\begin{tikzcd} V \arrow[r] \arrow[d] & W \\ \ell^\infty \arrow[r, "(\lambda_n)"] & \ell^1 \arrow[u] \end{tikzcd}$$

In addition, the distinction between Banach spaces and Smith spaces disappear, since

$$\begin{tikzcd} \ell^\infty_{Banach} \arrow[r, "(\lambda_n)"] \arrow[d] & \ell^1_{Banach} \arrow[d] \\ \ell^\infty_{Smith} \arrow[r, "(\lambda_n)"] \arrow[ur, "(\lambda_n)"] & \ell^1_{Smith} \end{tikzcd}$$

.

Rmk Any p-summable map is automatically p′-summable for p′ > p, and composite of p-summable and q-summable is r-summable, if $1/r = 1/p + 1/q$. So with a sufficient finite number of composable trace-class maps, can move through whole scale

$$0 < p, q < \infty :$$

Commensurable after finite number of compositions.

Def (Grothendieck) A dual nuclear Fréchet space is a countable filtered union of Banach spaces along trace-class injections.

A nuclear Fréchet space is a countable cofiltered limit of Banach spaces along dense trace-class maps.

(May view inside Cond($\mathbb{R}$).)

By above discussion, get the same classes of DNF and NF spaces, if we replace Banach by p-Banach for any $0<p<\infty$.

Claim If V, W are DNF spaces, then the p-liquid tensor product agrees with Grothendieck's completed tensor product,

$$V \otimes^{Liq_p} W \simeq V \hat{\otimes} W.$$

(Use $V = \bigcup_{n \in \mathbb{N}} \ell^{<p}_{Smith}$.)

Rmk Same is true for NF spaces, but proof is harder, since ε-δ's appear. Will try to stick to DNF spaces when developping complex analysis.

Ex Rings of holomorphic variables. (Will stick to 1-dimensional case for simplicity.) Will work liquid complex vector spaces. Define these as $\mathbb{C}$-modules in liquid real vector spaces.

$\bar{D} \subset \mathbb{C}$: closed unit disc.

Define $\mathcal{O}(\bar{D})$ to be the ring of functions, holomorphic on some open disc containing $\bar{D}$. It is a DNF space,

$$\mathcal{O}(\bar{D}) = \bigcup_{\varepsilon > 0} \{ (c_n) \mid \Sigma |c_n| (1+\varepsilon)^n < \infty \}.$$

For each ε, this is a sequence space, and transition maps are given by $(1, t, t^2, \ldots)$ for suitable t, so trace-class. In this case,

$$\mathcal{O}(\bar{D}_1) \otimes^{\mathrm{liq}, L} \mathcal{O}(\bar{D}_2)$$

$$\simeq \mathcal{O}(\overline{D_1 \times D_2}).$$

Cor Let $\bar{D}, \bar{D}' \subset \mathbb{C}$ be two closed discs, contained in a larger closed disc $\bar{E} \subset \mathbb{C}$. Then

$$\mathcal{O}(\bar{D}) \otimes^{Liq_p, L}_{\mathcal{O}(\bar{E})} \mathcal{O}(\bar{D}')$$

$$\simeq \mathcal{O}(\overline{D \cap D'}).$$

Pf First replace $\mathcal{O}(\bar{E})$ by $\mathbb{C}[T]$, calculate

$$\mathcal{O}(\bar{D}) \otimes^{L}_{\mathbb{C}[T]} \mathcal{O}(\bar{D}')$$

$$\simeq \mathcal{O}(\bar{D}) \otimes^{L} \mathcal{O}(\bar{D}') / (T_1 - T_2)$$

$$\simeq \mathcal{O}(\overline{D \times D'}) / (T_1 - T_2)$$

$$\simeq \mathcal{O}(\overline{D \cap D'}).$$

Special case,

$$\mathcal{O}(\bar{E}) \otimes^{L}_{\mathbb{C}[T]} \mathcal{O}(\bar{E}) \simeq \mathcal{O}(\bar{E}),$$

which implies that

$$- \otimes^{L}_{\mathcal{O}(\bar{E})} - \simeq - \otimes^{L}_{\mathbb{Z}[T]} - .$$

//

In particular, if $\bar{D} \cap \bar{D}' = \emptyset$, then

$$\mathcal{O}(\bar{D}) \otimes^{L}_{\mathcal{O}(\bar{E})} \mathcal{O}(\bar{D}') \simeq 0,$$

so algebra reflects topology $/\mathbb{C}$.

Wish to reprove two classical theorems:

$\underline{\text{Thm}}$ Let $X/\mathbb{C}$ be a compact Riemann surface, and let E/X be a holomorphic vector bdl.

1) For all $i \geq 0$, $H^i(X, E)$ is a f.d. $\mathbb{C}$-v.s., and for $i \gg 0$, it is zero.

2) Serre duality holds:

$$H^0(X, E)^\vee \simeq H^1(X, \Omega^1_{X/\mathbb{C}} \otimes E^\vee).$$

How to prove this using liquid theory? Produce a theory of quasi-coherent sheaves on cpt. complex manifolds equipped with a six-functor formalism. This makes it easy to prove duality statements locally.

1) will come from: Claim:

$\mathrm{Perf}(\mathbb{C}(*))$

$\simeq \mathrm{Nuc}_{\mathrm{Liqp}}(\mathbb{C}) \cap \mathrm{Cpt}_{\mathrm{Liqp}}(\mathbb{C})$.

Will see that $RP(X, E)$ is both nuclear and cpt, so 1) ensues. Will prove claim later.

2) Can work locally, but:

<u>Warning</u>: A theory of qcoh. sheaves on complex analytic spaces <u>has</u> to be derived. (There is no t-structure and no abelian cat. of qcoh. shv.)

<u>Reason</u>: For $D, D' \subset E$,

$$- \otimes_{\mathcal{O}(E)} \mathcal{O}(D)$$

is not exact.

<u>Pf</u> The restriction map

$$\mathcal{O}(E) \longrightarrow \mathcal{O}(D')$$

is injective, but if $D \cap D' = \emptyset$, then $- \otimes_{\mathcal{O}(E)} \mathcal{O}(D)$ takes it to

$$\mathcal{O}(D) \longrightarrow 0,$$

so $\mathcal{O}(E) \to \mathcal{O}(D)$ not flat. //

So it is just a fact of life that the t-structure gets messed up when you localize, so there is no global abelian category. It is not the end of the world, just a remark that you need to take into account. (Work ∞-categorically instead of derived 1-categorially.) /

We produced a six-functor formalism on solid quasi-coherent and coherent coh. in algebraic geometry (c.f. Condensed. pdf) and generalized to idea of "analytic spaces" and new idea of open subset called "steady localization." So even for schemes have new underlying "topological space" with many more open subsets and can descend along such opens.

In this lecture, will take a half-classical approach and use the classical underlying topological space, but equip it with a sheaf of condensed rings on it. Will build on the "original" six-functor formalism on

$$X \longmapsto \mathrm{Shv}(X, D(\mathbb{Z})),$$

where X is a f.d. locally compact Hausdorff space.

[In fact, Grothendieck first developed the six-functor formalism for étale sheaves in algebraic geometry, but that was a historical accident. Whence the quotation marks.]

For X as above,

$$D(\mathrm{Shv}(X, \mathbb{Z})) \simeq \mathrm{Shv}(X, D(\mathbb{Z})),$$

but RHS is much simpler to work with: $\mathrm{Shv}(X, \mathbb{Z})$ does not have enough projectives, so must derive using injectives, which obscures geometric intuition. Also, for X as above, $\mathrm{Shv}(X, D(\mathbb{Z}))$ is already hypercomplete and even Postnikov complete. Another reason to favor $\mathrm{Shv}(X, D(\mathbb{Z}))$ over $D(\mathrm{Shv}(X, \mathbb{Z}))$ is that we will want to replace ab. grps. by p-liquid modules, but $\mathrm{Liq}_p(\mathbb{C})$ does not have enough injectives.

Now for $f : Y \to X$ cts., have adjunction

$$\mathrm{Shv}(X, D(\mathbb{Z})) \underset{f_*}{\overset{f^*}{\rightleftarrows}} \mathrm{Shv}(Y, D(\mathbb{Z}))$$

with

$$(f_* \mathcal{G})(U) = \mathcal{G}(f^{-1}(U))$$

and with f^* given by the sheafification of the left Kan extension. In particular,

$$(f^* \mathcal{F})_y \simeq \mathcal{F}_{f(y)} .$$

Even though f_* is easier to define, it is harder to understand: it does not pres. colimits.

For $p : X \to *$,

$$p_* p^*(\mathbb{Z}) \in D(\mathbb{Z})$$

is a complex calc. the cohomology of X with (constant) $\mathbb{Z}$-coeff. So might expect that for a general map $f : X \to S$, f_* is the "in families" analogue of cohomology. But it is not!

<u>Thm</u> If $f : Y \to X$ is proper, then for every cart. square

$$\begin{array}{ccc} Y' & \xrightarrow{g'} & Y \\ \downarrow{\scriptstyle f'} & & \downarrow{\scriptstyle f} \\ X' & \xrightarrow{g} & X \end{array},$$

the canonical map

$$g^* f_* \longrightarrow f'_* g'^*$$

is an equivalence.

In particular, for $f : Y \to X$ proper, stalks of f_* is coh. of fibers:

$$(f_* \mathcal{G})_x \simeq R\Gamma(f^{-1}(x), \mathcal{G}|_{f^{-1}(x)}).$$

This is not true in general, as the following example shows:

Ex Consider the inclusion

$$\mathbb{C}^* = \mathbb{C} \setminus \{0\} \xrightarrow{j} \mathbb{C}$$

as a map of locally compact Hausdorff. In this case,

$$(j_* j^* \mathbb{Z})_0 \simeq \operatorname*{colim}_{U \ni 0} (j_* j^* \mathbb{Z})(U)$$

$$\simeq \operatorname*{colim}_{U \ni 0} (j^* \mathbb{Z})(f^{-1}(U))$$

$$\simeq \operatorname*{colim}_{U \ni 0} R\Gamma(U \setminus \{0\}, \mathbb{Z})$$

$$\simeq \mathbb{Z}[0] \oplus \mathbb{Z}[-1],$$

reflecting the coh. of $\mathbb{C}^*$ close to 0. In particular, it does not agree with cohomology of $j^{-1}(0) = \emptyset$. //

So f_* is bad, and $f_!$ is designed such that $f_! \simeq f_*$ if f is proper but such that $f_!$ satisfies base-change w.r.t. any map $g: X' \to X$ and s.t.

$f_!$ pres. colimits. Hence, it has a right adjoint $f^!$. In fact, it is easy to write down what $f_!$ and $f^!$ are explicitly. So let $f: Y \to X$ and let $\mathcal{F} \in \mathrm{Shv}(X, D(\mathbb{Z}))$ and $\mathcal{G} \in \mathrm{Shv}(T, D(\mathbb{Z}))$.

$$(f_! \mathcal{G})(U)$$

$$\simeq \operatorname*{colim}_{\substack{K \subset T \\ \text{proper} \downarrow \quad \downarrow f \\ U \subset X}} \mathrm{fib}(\mathcal{G}(T) \to \mathcal{G}(T \smallsetminus K))$$

$$(f^! \mathcal{F})(V)$$

$$\simeq R\mathrm{Hom}(f_!(\mathbb{Z}_V), \mathcal{F}).$$

The six-functor formalism is good for duality, e.g.

<u>Thm</u> (Poincaré-Verdier) Let M be a topological manifold, and let $p: M \to *$ be the unique map. For all $V \in D(\mathbb{Z})$,

$$p^!(V) \simeq p^*(V) \otimes p^!(\mathbb{Z}),$$

and $p^!(\mathbb{Z})$ is $\mathrm{Or}[d]$, where

Or is the orientation sheaf (or $\mathbb{Z}$ if M is orientable) and $d = \dim(M)$.

Pf For $j : U \hookrightarrow M$ the incl. of an open subset, $j^* \simeq j^!$, so $p^! \mathbb{Z}$ localizes on M. Check on $\mathbb{R}^d$ that $p^! \mathbb{Z} \simeq \mathrm{Or}[d]$, so done. (Calc.

$$R\Gamma_c(\mathbb{R}^d, \mathbb{Z}) \simeq \mathbb{Z}[-d].)$$

How does this impl classical Poincaré duality? By definition,

$$p_! p^*(\mathbb{Z}) \simeq R\Gamma_c(M, \mathbb{Z}),$$

and

$$R\underline{\mathrm{Hom}}(p_! p^*(\mathbb{Z}), \mathbb{Z})$$

$$\simeq R\underline{\mathrm{Hom}}(\mathbb{Z}, p_* p^!(\mathbb{Z}))$$

$$\simeq p_* p^!(\mathbb{Z})$$

thm

$$\simeq p_* p^* \mathrm{Or}[d]$$

$$\simeq R\Gamma(M, \mathrm{Or}[d]).$$

Since locally compact Hausdorff spaces are locally compact, sheaves should also be determined by their values on compact subsets. Indeed, this is the case and was first expressed by Lurie as follows:

$$\mathrm{Shv}(X, D(\mathbb{Z}))$$

$$\hookrightarrow \mathrm{Shv}\left(\begin{matrix}\text{site of cpt.} \\ \text{subsets of } X\end{matrix}, D(\mathbb{Z})\right)$$

finitary top.

is fully faithful w/ essential image the sheaves $\mathcal{G}$ s.t. the "over-convergence" cond.

$$\mathcal{G}(K) \simeq \operatorname*{colim}_{K \subset K'} \mathcal{G}(K')$$

is satisfied. The inclusion functor takes $\mathcal{F} \in \mathrm{Shv}(X, D(\mathbb{Z}))$ to the sheaf on cpt. subsets given by

$$\mathcal{F}(K) \simeq \operatorname*{colim}_{U \supset K} \mathcal{F}(U)$$

and it has a right adjoint that to $\mathcal{G} \in \mathrm{Shv}(\text{compacts}, D(\mathbb{Z}))$ assigns $\mathcal{F} \in \mathrm{Shv}(X, D(\mathbb{Z}))$ given by

$$\mathcal{F}(U) \simeq \lim_{\substack{K \subset U \\ \text{cpt}}} \mathcal{G}(K).$$

This point of view is more convenient in complex analytic setting, since can work with $\bar{D}$ instead on D.

So let X be a Riemann surf. (for simplicity). For $U \subset X$ open, let $\mathcal{O}(U)$ be the ring of holomorphic functions on U, viewed as a condensed (or even liquid) $\mathbb{C}$-alg. So maps

$$S \longrightarrow \mathcal{O}(U)$$

are in 1-1 correspondence w/ continuous maps

$$S \times U \longrightarrow \mathbb{C}$$

s.t. for all $s \in S$, the comp.

$$\{s\} \times U \hookrightarrow S \times U \longrightarrow \mathbb{C}$$

is holomorphic. For $\bar{D} \subset X$, the general formula for $\mathcal{O}(\bar{D})$ recovers what we had before, namely,

$$\mathcal{O}(\bar{D}) \simeq \operatorname{colim}_{U \supset \bar{D}} \mathcal{O}(U).$$

Consider sheaves of derived liquid $\mathbb{C}$-modules on X w/ an action of $\mathcal{O}(-)$.

<u>Def</u> Such an $\mathcal{O}$-module sheaf $\mathcal{M}$ is quasi-coherent if for every inclusion $\bar{D} \subset \bar{D}'$ of closed discs, the induced map

$$\mathcal{O}(\bar{D}) \otimes^{L}_{\mathcal{O}(\bar{D}')} \mathcal{M}(\bar{D}') \longrightarrow \mathcal{M}(\bar{D})$$

is an equivalence.

For holomorphic maps

$$Y \xrightarrow{f} X,$$

can define functor $f_!$ as in the "original" case. It will send $\mathcal{O}$-modules to $\mathcal{O}$-modules and preserve quasi-coherence.

For S extremally disconnected

$$(f_! \mathcal{G})(S) \simeq f_!(\mathcal{G}(S)),$$

so $f_!$ preserves colimits and has a right adjoint $f^!$

Key property:

Thm $p^!(V) \simeq p^*(V) \otimes p^!(\mathbb{C})$

and $p^!(\mathbb{C}) \simeq \Omega^1_{X/\mathbb{C}}[1]$.

Key: Understand $QCoh(D)$ for D and open disc. Claim: This is the full subcat. of $QCoh(\bar{D})$ spanned by those M s.t. $M \otimes_{\mathcal{O}(\bar{D})} \mathcal{O}(\bar{D} \setminus D) \simeq 0$.

Functors

$$QCoh(D) \underset{j^!}{\overset{j_!}{\rightleftarrows}} QCoh(\bar{D})$$

given by (for $\bar{E} \subset D$)

$$j^!(N)(\bar{E}) \simeq N \underset{\mathcal{O}(\bar{D})}{\otimes} \mathcal{O}(\bar{E})$$

and

$$j_!(M) \simeq \mathrm{fib}\big(M(D) \to M(D) \underset{\mathcal{O}(\bar{D})}{\otimes} \mathcal{O}(\bar{D} \setminus D)\big).$$

To prove that $j_!$ identifies $\mathrm{QCoh}(D)$ with the stated full subcat of $\mathrm{QCoh}(\bar{D})$ follows from the tensor product calc. made earlier.

So $\mathrm{QCoh}(D)$ is obtained from $\mathrm{QCoh}(\bar{D})$ by killing an idempotent algebra. This leads to explicit descriptions of each of the functors

$$p_*,\ p^*,\ p_!,\ p^!$$

For example,

$$p^!(V)$$

$$\simeq \mathrm{RHom}_{\mathcal{O}(\bar{D})}\big(\mathrm{fib}(\mathcal{O}(\bar{D}) \to \mathcal{O}(\bar{D} \setminus D)),\ V \otimes \mathcal{O}(\bar{D})\big)$$

Reduced to calc.

$$\mathrm{fib}(\mathcal{O}(\bar{D}) \to \mathcal{O}(\bar{D} \setminus D))$$

which is a calc. with power series in one variable. The result is that the fiber is given by 1-forms on the complementary closed disc in $\mathbb{P}^1$.

Conclusion :

$$p^! \simeq p^* \otimes \Omega^1_{X/\mathbb{C}}[1].$$

In particular, $p^!$ preserves all colimits, i.e. the right adj. of $p_!$ preserves colimits, and this implies that $p_!$ sends compact objects to compact objects. But a vector bundle V is dualizable and therefore, it is compact if and only if $\mathcal{O}$ is compact, which, in turn, happens if and only if p_* preserves colimits. So if p is proper (i.e. X cpt.), then

$$RP(X, V) \simeq p_*(V) \simeq p_!(V)$$

is a compact obj. in $\mathrm{Liq}_p(\mathbb{C})$. On the other hand, it is, by definition, a finite limit of $\mathcal{O}(\bar{D})^{\oplus n}$, which implies that it is nuclear in the sense to be discussed below. By

nuclear + compact

= perfect,

conclude that $R\Gamma(X, V)$ is perfect, i.e. "finiteness of coh." Also, get Serre duality as in Verdier setting.

So the liquid framework produces these basic theorem's (which imply e.g. that every compact Riemann surface is algebraic) to the one case of the fiber ($\simeq$ cokernel) of

$$\mathcal{O}(\bar{D}) \longrightarrow \mathcal{O}(\bar{D} \setminus D).$$

Nuclearity: Define $M \in D(\mathrm{Liq}_p(\mathbb{C}))$ to be nuclear if for all S extremally disconn., the map

$$(C(S,\mathbb{C}) \otimes^L M)(*) \longrightarrow M(S)$$

is an equivalence.

Rmk Two differences from p-adic situation: 1) The Banach space $C(S,\mathbb{C})$ is almost certainly not flat, so must work with derived tensor product. 2) Do not know that this definition of nuclearity is closed under passage to $\pi_i(-)$, so do not have an abelian category of nuclear modules.

So for foundational reasons, must work derived. But this does not give any complications for applications to complex analytic geometry, since tensor products and derived tensor products agree for NF and DNF spaces.

The full subcat. of $D(\mathrm{Liq}_p(\mathbb{C}))$ spanned by the nuclear derived p-liquid $\mathbb{C}$-v.s. is gen. under colimits (in fact freely gen. under sufficiently filtered colim.)

by the "basic nuclear" obj. of the form

$$\mathrm{colim}\,(M_0 \to M_1 \to M_2 \to \cdots)$$

with M_i cpt. proj. and with maps trace-class. Among these are the DNF, which corresp. to the basic nuclear obj., where the maps are also inj.

Since being nuclear is stable under all colimits, this justifies that $RP(X, V)$ is nuclear.

Recall also that

dualizable = nuclear + compact

so enough to see that

perfect = dualizable.

Now, the full subcategory of dualizables is generated (in what sense?) by objects of the form

$$\mathrm{cone}\,(M \xrightarrow{id - f} M)$$

with M compact projective and with f trace-class. So must prove the liquid analogue of the basic fact in functional analysis that for f compact oper., id $- f$ is Fredholm. In particular need to show that

$$\mathrm{cone}\,(M \xrightarrow{\mathrm{id}-f} M)$$

is discrete. In the p-adic case, there is an algebraic argument for this. In the liquid situation, use that this object is dualizable, and that its dual is of the form

$$\mathrm{cone}\,(V \xrightarrow{\mathrm{id}-g} V)$$

with V Banach and g compact. So cone is perfect, by classical fact, and hence, so is its dual.