

CLAUDIA SCHEINBAUER 7

EXTRA REFERENCE AND AUTHORS FOR FACTORIZATION HOMOLOGY

- o) MORRISON - WALKER BLOB HOMOLOGY
- o) KNUDSEN } CONFIGURATION SPACES
- o) QUOCHO }
- + Alg Geo vs Affine
BEILINSON - DRINFELD
- o) KUPERS - MINER
- KRANNICH
- o) KLANG, BAUDKLAYDER
- o) KIRILOV - YING HONG THOM.

DUALIZABILITY

DEF: A FULLY EXTENDED TROTTED n -di TOPOLOGICAL FIELD THEORY IS A SYN. MON. FUNCTOR OF (∞, n) -CATEGORIES $\text{Bord}_n^{\text{fr}} \rightarrow \mathcal{C}$
SYN. MON

Cobordism Hypothesis: (BAEZ-DOLAN, LURIE (SKETCH OF PROOF))
 $n=1$ PROOF BY HARPAZ (FOR $(\infty, 1)$ -CATS)
 AYALA-FRANCIS (-ROSENBLYUM) - UNFINISHED
 SCHOMMER - PRIED ...

↑
 ??? NEED TO CHOOSE A "NICE" TARGET.

$\text{Fun}^{\otimes}(\text{Bord}_n^{\text{fr}}, \mathcal{C}) \xrightarrow[\cong]{\text{EV}_{\pm} \approx \mathbb{R}^n} (\mathcal{C}^{n\text{-DUALIZABLE}})^{\approx}$ IS AN EQUIV OF ∞ -GROUPS.

↑
Follows from Bord_n HAVING DUALS

∞ -GROUPOID

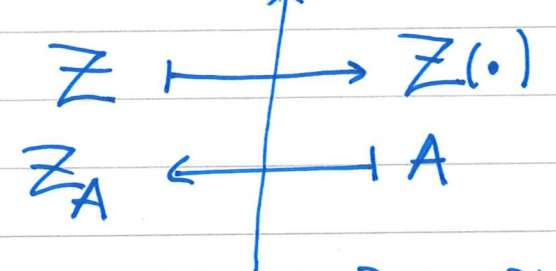
EXPRESSED LOCALITY.

For $\mathcal{C} = \text{Alg}_n(\mathcal{J})$:

THM (GUILIUM-S)^(A) EVERY OBJECT IN $\text{Alg}_n(\mathcal{J})$ IS n-DUALIZABLE.

$\rightsquigarrow \text{Fun}^{\otimes}(\text{Bord}_n, \text{Alg}_n(\mathcal{J})) \cong (\text{Alg}_n(\mathcal{S}))^{\sim}$

Cobordism Hypothesis becomes

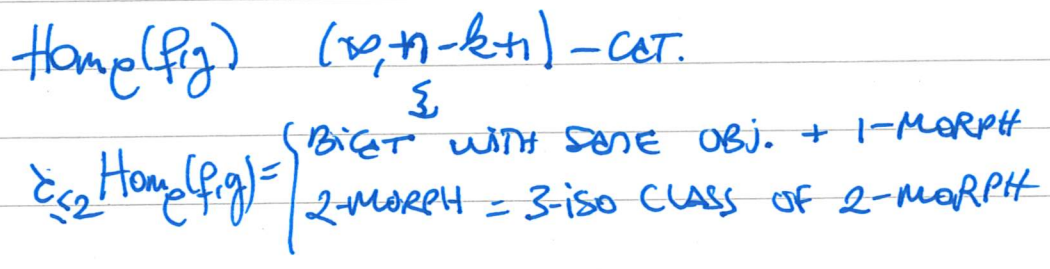
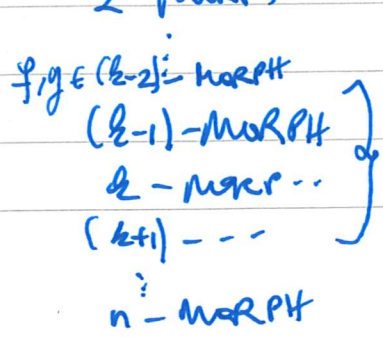
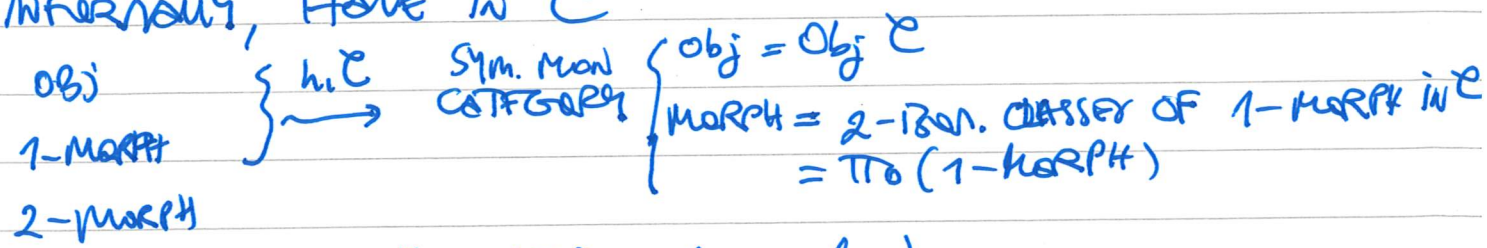


SHOULD BE ABLE TO CHECK THE COBORDISM HYP. IN THIS CASE.

DEF: AN OBJECT X IN A SYM. MON (∞, n)-CAT \mathcal{C} IS n-DUALIZABLE IF [SHORT VERSION: AROUJO]

- (1) X HAS A DUAL (AS OBJECT IN $h_1\mathcal{C}$)
- (2) ev, coev HAVE LEFT AND RIGHT ADJOINT WITH UNITS AND COUNTS
- (3) --- KEEP GOING UNTIL WE GET TO n .

INFORMALLY, HAVE IN \mathcal{C}



DEF: LET \mathcal{D} BE A BICATEGORY ($\mathcal{D} = \text{Cat}$)

A 1-MORPH $R: c \rightarrow d$ HAS A LEFT ADJOINT IF $\exists L: d \rightarrow c$ AND 2-MORPH $c: L \circ R \Rightarrow \text{id}_c$ S.T. $u: \text{id}_d \Rightarrow R \circ L$

- (1) $R = \text{id}_d \circ R \xrightarrow{u \circ \text{id}_R} R \circ L \circ R \xrightarrow{\text{id}_R \circ c} R$ IS id_R
- (2) $L \Rightarrow L \circ R \circ L \Rightarrow L$ IS id_L .

A k -MORPHISM α IN \mathcal{C} HAS A LEFT AND RIGHT ADJOINT IF IT DOES IN $\mathcal{Z}_{\leq 2} \text{Hom}_{\mathcal{C}}(f, g)$

NOTE: IN n -REALIZABILITY, (1) USED A SYM. MONOIDAL 1-CATEGORY AND THAT COULD BE "DEWOOPED" TO A BICATEGORY. FROM (2) ON, WE HAVE NO MONOIDAL STRUCTURE.

(n, k) -CAT
↓

OTHER TARGETS: • $\text{Alg}_n(\mathcal{C})$ IS $(n, n+k)$ -CAT

~> CAN DO $\text{Alg}_1(\text{Alg}_1(\dots (\text{Vect})))$

- TAKE LFP, $\text{Alg}_1(\text{LFP}), \dots$
- BICATEGORY TARGETS: SEE APPENDIX OF A PAPER OF BARTUET-BOUGLAS-SHLOMER-PIERRE-VICARY -
- UNIVERSAL TARGET ??

• INJECTIVE TFTS HAVE A UNIVERSAL TARGET (FREE) + ...

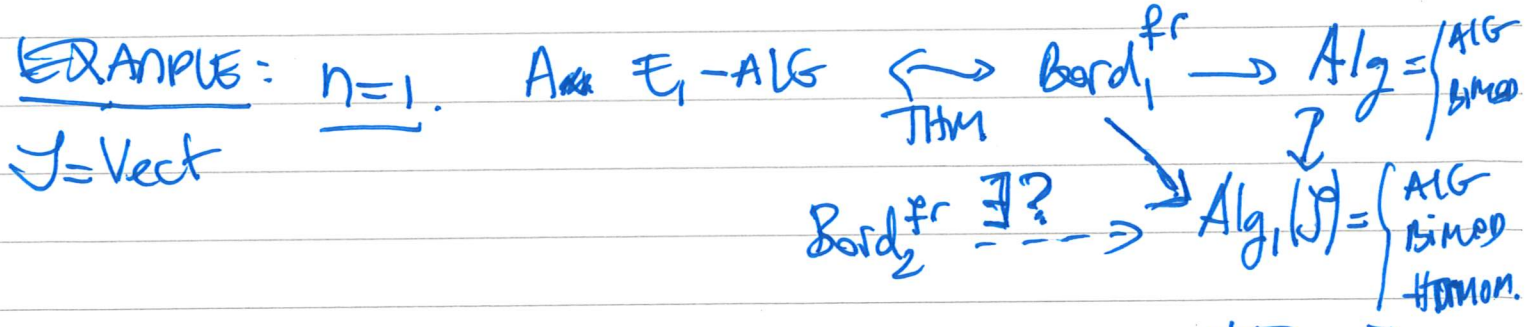
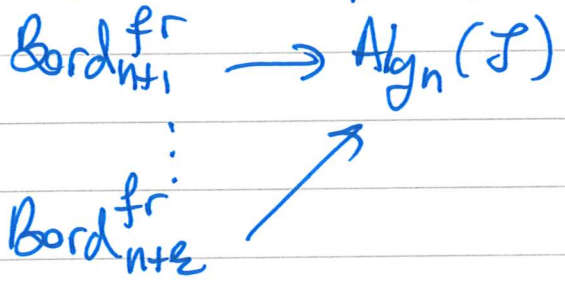
For $\mathcal{C} = \text{Bord}_n^{\text{fr}}$, GET $(\text{Bord}_n^{\text{fr}})^{\cong} = \text{UNBOUNDED GROUPS}$.

QUESTION (GHOST) IS $\text{Alg}_n(\text{MFD}_n^{\text{fr}}) \cong \text{Bord}_n^{\text{fr}}$??
 WHAT ARE E_n -ALG IN FRAMED n -MFDs ?

PROBLEM [GG] (B) IN $\text{Alg}_n(\mathcal{J})$ USING FACT ALG, IF A IS $(n+1)$ -DIAGONALIZABLE, THEN $A \cong \mathbb{1}$.

REASON: POINTINGS ! FACTORIZATION ALGEBRA FROM $\emptyset \subseteq U$.

SO WOULD NEED TO MODIFY $\text{Alg}_n(\mathcal{J})$ OR USE HAUSSERNG + JOHNSON-FREYD-S TO GET POSSIBLY $(n+1)$, ..., $(n+k)$ DIAGONALIZABILITY, AND GET



WE DON'T SEE THOSE!

$n=2$, $\mathcal{J} = \text{Cat}$ or LFP, OBJ. OF $\text{Alg}_2(\text{LFP})$

THM $(G-S) \Rightarrow A$ is 2-DENORMALIZABLE

COB. HYP \leadsto 2d TFT Bord₂^{fr JA} \rightarrow $\text{Alg}_2(\text{LFP})$

ARE BRAIDED MAN BICAT \rightarrow Rep \mathcal{B} , Rep \mathcal{G}

\downarrow
COMPUTE FURTHER HOMOLOGY

$$\int_A \Sigma$$

QUESTION: $\text{Alg}_2(\text{LFP})$ is (2,4)-CAT.

CAN ONE GET A 3-d TFT OR 4-d TFT ?

ANSWER: YES, if ... A RIGID \Rightarrow 3-DENORMALIZABLE

BRAIDED FUNCTOR \Rightarrow 4-DENORMALIZABLE

in MODIFIED Alg_2 (UNPOINTED VERSION)

(OTHERWISE WOULD HAVE TO BE TRIVIAL)

[BRACKETING - JORDAN - SYMBOLE]

QUESTION: WHAT DO THESE 3d / 4-d TFT'S LOOK LIKE ??

In 3-d, SHOULD GET THE SKEW MODULE ...

NOTE: SOMETIMES THE COB. HYPOTHESIS IS NOT THE RIGHT FRAMEWORK TO WORK WITH ... THERE ARE OTHER THINGS OUT THERE!