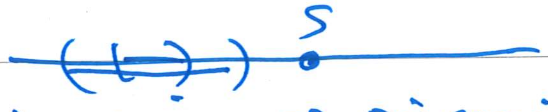
CLAUDIA SCHEIMBAUER, LECTURE 6

BIMODULE EXAMPLE:



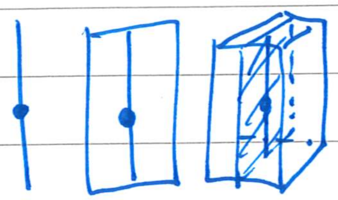
inclusion of discs in same stratum induce equiv.

CONSTRUCTIBLE ALGEBRA ON $(0,1)^n = X_n \supset \dots \supset X_0 \supseteq X_{-1} = \emptyset$

WHERE •) X_i i-dim. smooth submfd of $(0,1)^n$

•) X_i finitely many conn. components

•) every point $x \in X_n$ has open nbhd



diffeo to "standard stratified disk", i.e.

$$(0,1)^n \supseteq \{\tfrac{1}{2}\}\times(0,1)^{n-1} \supseteq \dots \supseteq \{(\tfrac{1}{2},\dots,\tfrac{1}{2})\}\times(0,1)$$

is a factorization alg on $(0,1)^n$ s.t. any inclusion of discs $D_1 \hookrightarrow D_2$ where both are diffeo to the same standard disk, the struct. $\mathcal{F}(D_1) \xrightarrow{\simeq} \mathcal{F}(D_2)$

Def: • $p: X \to Y$ cont. map, $\mathcal{F}$ fact. alg on X

$\Rightarrow p_*\mathcal{F}(U) := \mathcal{F}(p^{-1}(U))$ is a fact. alg on Y.

Rem: using the descent condition, one can extend a factorization alg to all open subsets in M.

• If p is a proper submersion, $\mathcal{F}$ loc. constant

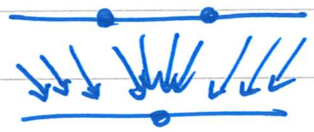
$\Rightarrow p_*\mathcal{F}$ is locally cst

(i.e. fiber bdl)

IF $\mathcal{F}$ is CONSTRUCTIBLE, AND p is

- LOCAL DIFFEO
- REFINEMENT OF STRATIFICATION $\Longrightarrow p_*\mathcal{F}$ CONSTRUCTIBLE.
- COLLAPSE AND RESCALE MAPS

eg



THE TARGET HIGHER CATEGORY

RECAP: WANT $\mathrm{Bord}_n^{fr,(\infty,n)} \xrightarrow{\int A} \mathcal{C} = \mathrm{Alg}(\mathcal{S})$

SYMM. MON. FUNCTOR OF (∞,n)-CATEGORIES

= FULLY EXTENDED FRAMED n TFT

INFORMALLY: OBJ = POINTS $\longleftrightarrow \coprod_{\varepsilon} \mathbb{R}^n$, FRAMING

1-MORPH = FRAMED 1-BORDISMS

$\vdots$

n-MORPH = n-BORDISMS

$(n+1)$-MORPH = DIFFEO

$\vdots$

EX: $n=1$, $\mathcal{S} = \mathrm{Vect} \rightsquigarrow \mathcal{C} = \mathrm{Alg}$

$n=2$, $\mathcal{S} = \mathrm{CAT} \rightsquigarrow \mathcal{C}$ SEE ADRIEN'S LECTURES (BRAIDED MON. CATS)

~~[illegible]~~ A E_n-ALG IN $\mathcal{S}$ = LOCALLY CONST. FACTORIZATION ALG ON $(0,1)^n$. $\rightarrow \int_{\coprod_{\varepsilon} \mathbb{R}^n} A$

FOR HIGHER MORPHISMS, NEED A CONSTRUCTIBLE FACT. ALG ON $(0,1)^n$ ...

$\mathrm{Alg}_n(\mathcal{S})$: • VERSION USING GEN. ∞-OPERADS [HAUGSENG]

$(\infty, n+1)$-CATS • USING FACT. ALG [CALAQUE-S]

FOR $\mathcal{S}$ (∞, k)-CAT $\rightsquigarrow$ $\mathrm{Alg}_n(\mathcal{S})$ $(\infty, n+k)$-CAT [JOHNSON-FREYD-S] USING EITHER ABOVE CONST.

$(\infty, n+1)$-CAT = (COMPLETE) n-FOLD SEGAL $(\infty,1)$-CAT.

WE WILL LOOK AT $\mathrm{Alg}_1(\mathcal{S})$ AS SEMI-SIMPLICIAL $(\infty,1)$-CAT. INFORMALLY:

X_0 = {LOC. CONST. FACT ALG ON $(0,1)$} in $(\infty,1)$-CAT

d_0/d_1 RESTRICT TO LHS/RHS

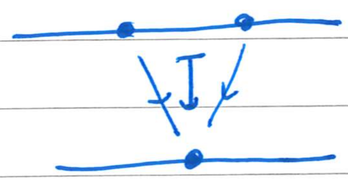
X_1 = CONSTRUCTIBLE FACT ALG ON s

X_2 = " s_1 s_2

$\vdots$

d_0/d_2 RESTRICT TO LEFT/RIGHT

d_1



PUSHFORWARD ALONG THE COLLAPSE.

CLAIM: THIS SATISFIES THE SEGAL CONDITION.
(TRUE BECAUSE FACTORIZATION ALGS GLUE)

NOTE: THE DEGENERACIES ARE A LITTLE TROUBLESOME... NEED TO REPLACE POINTS BY INTERVALS ETC.

NOW ... THE (EXTENDED) TFT

$$Z_A : \mathrm{Bord}_n^{(\infty,1)} \longrightarrow \mathrm{Alg}_1(\mathcal{S})$$

GIVEN $A : \mathrm{Disk}_n^{fr} \to \mathcal{S}$

$\Downarrow$

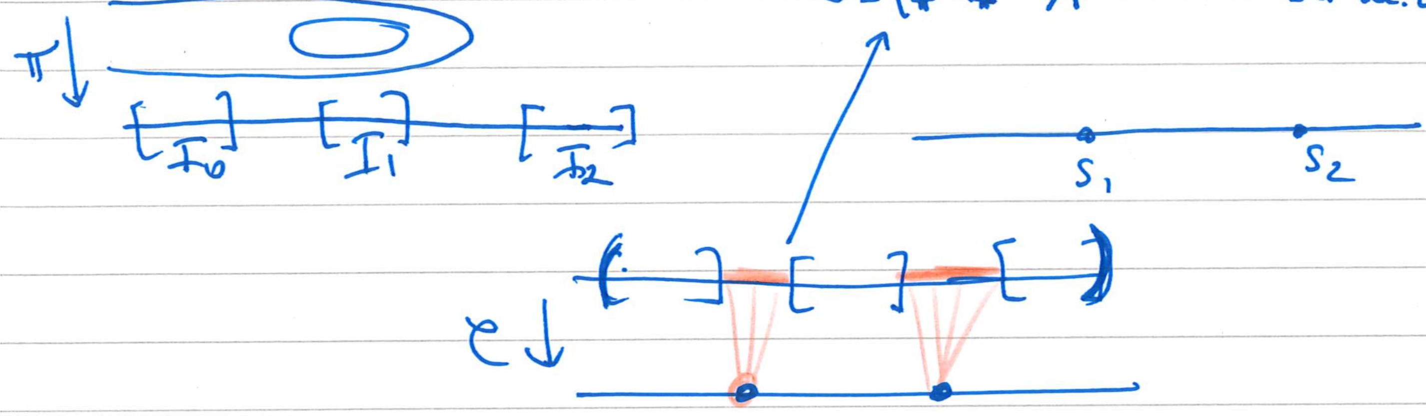
$(M, I_0 \le \dots \le I_\ell) \rightsquigarrow \mathcal{F}_A$ ON M : $U \mapsto \int_U A \rightsquigarrow \pi_* \mathcal{F}_A$

(LOC. CONST)

(NOT LOC. CST) ON $(a_0, b_\ell) \simeq (0,1)$ BUT LOC. CST ON I_i's

$\wr$

$\mathcal{F} = \rho_* \pi_* \mathcal{F}_A$



<u>LEMMA</u>: $\mathcal{F}$ is CONSTRUCTIBLE.

<u>EX</u>: $\mathcal{F}_A$ $A = (E_1-)\mathrm{ALG}$