Module Example: \( \frac{(-\frac{1}{3})^5}{5} \)  
Inclusion of disc in same stratum induce equiv.

Constructible algebra on \((\mathbb{R}, 1)^n = X_n\) \(\rightarrow\) \(\mathbb{R}^2 \times \mathbb{R} = 0\)

Where  
1) \(X_i\) i-di smooth subhed of \((\mathbb{R}, 1)^n\)
2) \(X_i\) finitely many conn. components
3) every point \(x \in X_n\) has open nbhd diffeo to "standard stratified disk", i.e.
\[
(91) \quad \frac{1}{2} \cdot \frac{1}{3} \cdot (\frac{1}{2})^2 \cdot \cdots \cdot \frac{1}{2} \cdot \frac{1}{3} \cdot (\frac{1}{2})^2 \cdot \cdots
\]

is a factorization alg on \((\mathbb{R}, 1)^n\) s.t.
any inclusion of discs \(D \rightarrow D_2\) where both are diffeo to the same standard disk, the struct. \(F(D_1) \rightarrow F(D_2)\)

Def: \(\Phi: X \rightarrow Y\) cont. map, \(F\) fact. alg on \(X\)

\[ \Rightarrow p_\ast F(U) := F(p^{-1}(U)) \text{ is a fact. alg on } Y \]

Rem: using the descent condition, one can extend a factorization alg to an open subset in \(M\).

If \(p\) is a submersion \(F\) loc. constant

\[ \Rightarrow p_\ast F \text{ is locally } CST \]

(\text{fibers BLD})
If $J$ is constructible, and $p$ is

- Local diffed
- Refinement of stratification
- Collapse and rescue map

\[ \text{The target higher category} \]

Recap: \[ \text{Want } \text{ Bord}_n^{fr,(\infty,n)} \int_A \mathcal{C} = \text{Alg}(J) \]

Sym. Mon. Functor of $(\infty,n)$-categories

= Fully extended trapped n TFT

Informally: Obj = Points \[ \Leftrightarrow \] $U \in \text{T}^n$, Framing

- Morph = Framed $n$-Bordisms

- $n$-Morph = $n$-Bordisms
- $(n+1)$-Morph = Diffed

Ex: \[ n=1, J = \text{Vect} \to \mathcal{C} = \text{Alg} \]

\[ n=2, J = \text{CAT} \to \mathcal{C} \text{ is Adrien's lectures} \]

\[ \text{See (braided mon.) Cats} \]

$\mathcal{A}$-Alg in $J = \text{local} \ (\text{const.})$

Factorization are on $(0,1)^n \to \int \mathcal{A}$
For higher morphisms, need a constructible fact Alg on $(0,1)^n$.

\[ \text{Alg}_n(S) : \text{ vexendo using gen. do-operads} \]
\[ (\infty, n+1) - \text{ CAT} \rightarrow \text{ using fact Alg [calaque-s]} \]

For \( J : (\infty, 1) - \text{ CAT} \rightarrow \text{ Alg}_n(S) (\infty, n+1) - \text{ CAT} \)
\[ \text{ [johnson-freyd-s] using either above count.} \]

\( (\infty, n+1) - \text{ CAT} = \text{(compute) n-fold segal} \)
\[ \text{ (0,1) - CAT}. \]

We will look at \( \text{ Alg}_n(\infty) \) as semi-simplicial \( (0,1) - \text{ CAT} \).

Informally:

\[ \mathbf{X}_0 = \text{ loc. const. fact Alg on } (0,1)^n \]
\[ \mathbf{X}_1 = \text{ constructible fact Alg on } S \]
\[ \mathbf{d}_0/\mathbf{d}_1 \text{ restrict to left/right } \]
\[ \text{ push forward along the cocone.} \]

Claim: This satisfies the segal condition.

(True because factorizationAlg's glue)

Note: The degeneracies are a little troublesome... need to replace points by intervals etc.
Now... the (extended) TFT

\[ \mathcal{Z}_A : \text{Bord}_n \to \text{Alg}_1(\mathcal{F}) \]

Given \( A : \text{Dis}_k \to \mathcal{F} \)

\( (M, \mathcal{I}_0 \leq \mathcal{I}_1 \leq \mathcal{I}_2) \mapsto \mathcal{F}_A \text{ on } M : U \to \mathcal{F}_A \)

(Not loc. const.
Not loc. const.
On \( \mathcal{D}_0, \mathcal{D}_1 \)
But loc. const. on \( \mathcal{I}_2 \)

**Lemma:** \( \mathcal{F}_A \) is constructible.

**Ex:**

\[ \mathcal{F}_A \]

\[ A = (\epsilon_1-\epsilon_0)A(G) \]

\[ \pi_1 \mathcal{F}_A \]

\[ \mathcal{F}(\partial) = 1 \]

\[ A \otimes A^\text{op} \to A \]

Excision

\[ L \]

\[ \mathcal{F}_A \]

\[ A \otimes A \]

\[ A \otimes A^\text{op} \]

\[ 1 \]