SEIBERG-WITTEN INVARIANTS: CONTRIBUTED TALKS

1. Misha Temkin (University of Freiburg): *Combinatorial computation of Seiberg-Witten invariant of 3-manifolds*

Definition of a Seiberg-Witten invariant of a 4-manifold can be touched up to yield an invariant of a 3-manifold Y. Turaev and Meng-Taubes proved that this invariant can be computed "combinatorially", i.e. in terms of a cell decomposition of Y. The key ingredient is Milnor's torsion, which is a generalization of a determinant in the realm of chain complexes. We will outline the main notions of this combinatorial approach.

2. Andres David Ramirez Ruiz (University of Miami): *Instanton L-spaces for higher rank gauge groups*

We will explore the relation between instanton homology for higher rank gauge groups and $U(2)$ instanton homology. For any closed, oriented, connected 3-manifold, the Euler characteristics of such homology groups are known to be related. When seeking a direct relation between the homology groups, the first class of manifold one might try to say something about are Instanton L-spaces. We will explain this relation for some spherical 3-manifolds

3. Sebastián Matías Camponovo (Università di Trieste): *Exotic Definite 4-Manifolds with infinite fundamental group.*

We say that a 4-manifold X has an exotic smooth structure if X possesses more than one smooth structure, i.e., there exists a 4-manifold X' that is homeomorphic but not diffeomorphic to X. Although examples of exotic smooth structures on closed 4-manifolds have been known since the 1980s, the definite case remained elusive for decades.

The first examples of exotic smooth structures on 4-manifolds with definite intersection form were produced by Levine-Lidman-Piccirillo in 2023 their construction yielded manifolds with fundamental group $\mathbb{Z}/2$. Subsequently, Baykur-Stipsicz-Szabó and Harris–Naylor–Park developed related techniques to obtain examples with fundamental groups $\mathbb{Z}/(4k+2)$ and $\mathbb{Z}/2 \times \mathbb{Z}/2$, respectively.

The aim of this talk is to review the main ideas underlying these constructions and to explain how exotic phenomena can be produced via involutions on covering spaces. Finally, we will discuss how a related approach can be used to obtain examples with new fundamental groups, starting from appropriate exotic building blocks.

4. Jacek Rzemieniecki (Humboldt-Universität zu Berlin): Spin(7)*-manifolds, exotic Dehn twists and family Seiberg-Witten invariants*

In this talk, I will present an unexpected application of recent developments in families Seiberg-Witten theory to a problem in Spin(7) geometry. Spin(7)-manifolds are 8-dimensional Riemannian manifolds with holonomy equal to the exceptional Lie group Spin(7). They carry distinguished calibrated 4-dimensional submanifolds, called Cayleys, which play a role analogous to that of special Lagrangians in Calabi-Yau geometry. Donaldson proposed a programme to study fibrations of Spin(7)-manifolds by Cayleys, and more specifically by Cayleys of the topological type of a K3 surface. It has long been conjectured that such fibrations must necessarily include singular fibers. I will explain recent work showing that any such smooth fibration is reduced to two topological cases: the K3-fiber case is ruled out using gauge-theoretic input from families Seiberg–Witten theory, while the remaining case is tied to a question about boundary Dehn twists in 4-manifold topology. This is based on a recent preprint joint with Viktor Majewski.