# Geometry and Topology, ICM sectional workshop

**University of Copenhagen**

**6-14 July 2022**

## Titles and Abstracts of short communications

**Wednesday 6th, 18:00 - 20:00**

**Elisabeth Gasparim** - Mirror Symmetry for Adjoint Orbits
Abstract: I will describe Mirror Symmetry for Adjoint Orbits of sl(n) considered together with a complex potential with singularities of Morse type. The 2 sides of the mirror are obtained by calculating: A. The Fukaya–Seidel category of Vanishing Cycles, and B. The Orlov category of singularities of the dual.

**Jens Reinhold** - Simplicial volume and essentiality of manifolds fibered over spheres
Abstract: This talk concerns the question when a manifold that fibers over a sphere can be rationally essential, or even have positive simplicial volume. I will explain how to see that mapping tori of manifolds (whose fundamental groups can be quite arbitrary) of dimension $2n + 1 \geq 7$ with non-zero simplicial volume are very common. This contrasts the case of fiber bundles over a sphere of dimension $d \geq 2$ whose total spaces are rationally inessential if $d \geq 3$, and always have simplicial vanishing volume. Based on joint work with T. Kastenholz."

**Ian Hambleton** - A Stability Range for 4-Manifolds
Abstract: Two closed oriented 4-manifolds $M$ and $N$ are ""stably homeomorphic"" if they become homeomorphic after connected sum with copies of $S^2 \times S^2$. The talk would describe a new 4-dimensional stability range invariant, which provides estimates for how many copies of $S^2 \times S^2$ are actually needed to produce a stable homeomorphism.

**Robin Stoll** - The stable cohomology of self-equivalences of connected sums of products of spheres
Abstract: I will explain an identification of the cohomology of the stable classifying space of homotopy automorphisms (relative to an embedded disk) of connected sums of $S^k \times S^l$, where $2 < k < l < 2k - 1$. The result is expressed in terms of Lie graph complex homology.

**Peter Patzt** - Cohomology of arithmetic groups
Abstract: In this talk, I will give a short overview of the cohomology of arithmetic groups such as the special linear groups of the integers and their congruence subgroups. The low-degree cohomology has well-understood patterns that I will recall. For the high-degree cohomology much less is known but I will present some recent progress.

**Sil Linskens** - Global Spectra via Partially Lax Limits
Abstract: Global stable homotopy theory is a formalism which admits applications to orbifold cohomology, elliptic cohomology and importantly equivariant stable homotopy theory. Many important equivariant spectra admit global refinements, and these refinements have led to theoretical advances and have aided in calculations. However not only does global homotopy theory admit applications to equivariant homotopy theory, but it seems reasonable to expect that global stable homotopy theory is in some sense determined by equivariant stable homotopy theory. For example, there exists a restriction functor which sends a global spectrum to a G-spectrum for every compact Lie group G. Furthermore, these collectively form a conservative family of functors. I will present a result which explains exactly how the infinity category of global spectra is built from the infinity categories of G-spectra. It says that the infinity category of global spectra is a partially lax limit of a diagram indexing the infinity categories of G-spectra for all G, lax on the surjections and strict on the injections. (The work presented is joint with Denis Nardin and Luca Pol).

## Sunday 10th, 15:15 - 16:00

**Augustin Moreno** - A modern look into the 3-body problem
Abstract: Despite the fact that the 3-body problem is an ancient conundrum that goes back to Newton, it is remarkably poorly understood, and is still a benchmark for modern developments. In this talk, I will give a biased account of this classical problem, both from a modern theoretical perspective, i.e. outlining possible lines of attack coming from symplectic geometry, holomorphic curves and Floer theory; as well as comment on practical and numerical aspects, within the context of finding orbits for space mission design and ocean worlds exploration.

**Renee Hoekzema** - Cut-and-paste invariants of manifolds via algebraic K-theory
Abstract: Recent work of Jonathan Campbell and Inna Zakharevich has focused on building machinery for studying scissors congruence problems via algebraic K-theory. In this talk I present a new application of their framework to the study of cut-and-paste invariants of manifolds. We construct a spectrum that recovers the classical SK (schneiden und kleben, German for cut and paste) groups for manifolds on the zeroth homotopy group, and we construct a lift of the Euler characteristic, one of these invariants, to a map of spectra. This is joint work with M. Merling, L. Murray, C. Rovi and J. Semikina.

**David White** - The homotopy theory of higher operads
Abstract: I will give a brief overview of a recent paper, joint with Michael Batanin and accepted to Transactions of the AMS, that proves

a stabilization theorem for higher braided operads, and deduces Baez-Dolan stabilization as a corollary. We produce (semi) model structures for the category of k-operads and for higher braided operads, a machinery for left Bousfield localization without left properness, and a theory of homotopical Beck-Chevalley squares, all en route to proving the stabilization result.

**Monday 11th, 17:00 - 19:00**

**Vitaliy Kurlin** - Geometric Data Science challenges and solutions
Abstract: The talk reviews the advances in Geometric Data Science [1] studying computable and continuous metrics on finite and periodic sets of points considered up to rigid motion or more general isometry in any Euclidean or metric space. The persistent homology is an isometry invariant of finite point sets and turned out to be much weaker than previously anticipated [2]. The space of all 2D lattices up to rigid motion and uniform scaling is a 2D sphere without one point [3]. All 660K+ periodic crystals in the world's largest Cambridge Structural Database were distinguished by 200B+ pairwise comparisons of new invariants over two days on a modest desktop computer, establishing the Crystal Isometry Principle [4]: any real crystal is uniquely determined by its geometry of atomic centers. Hence all periodic crystals live in a common Crystal Isometry Space (CRISP) continuosuly extending Mendeleev's table of elements.
[1] O.Anosova, V.Kurlin. An isometry classification of periodic point sets. Proceedings of Discrete Geometry and Mathematical Morphology 2021.
[2] P.Smith, V.Kurlin. Families of point sets with identical 1D persistence. arxiv:2202.00577.
[3] V.Kurlin. Mathematics of 2-dimensional lattices. arxiv:2201.05150.
[4] D.Widdowson et al. Average Minimum Distances of periodic point sets. MATCH Communications in Math. Comp. Chemistry, v.87(3), p.529-559, 2022.

**Suraj Prakash Yadav** - $A^1$ connectivity of moduli stack of vector bundles in a curve
Abstract: I will talk about motivic connected component of $A^1$ connectivity of moduli stack of vector bundles in a curve with a fixed determinant and how such a result leads towards classification of projective bundles on a curve upto their motivic homotopy type

**Achim Krause** - On the K-theory of $\mathbb{Z}/p^n$
Abstract: In recent work with Antieau and Nikolaus, we develop an explicit algebraic description of $\mathbb{Z}/p^n$ based on prismatic cohomology. This description leads to an effective computer algorithm to compute arbitrary K-groups of $\mathbb{Z}/p^n$, but also to surprising general patterns: We prove that the K-groups of $\mathbb{Z}/p^n$ vanish in large even degrees, and give an explicit description of their orders in large odd degrees.

**Andrei Konovalov** - Semi-topological K-theory of dg-algebras and the lattice conjecture
Abstract: I will discuss the problem of constructing a natural rational structure on periodic cyclic homology of dg-algebras and dg-categories. The information given by A. Blancs topological K-theory, which provides such a structure in a series of examples, will be discussed, together with its structural properties and possible applications.

**Michael Roop** - Singularities of solutions to barotropic Euler's equations
Abstract: We will discuss a geometric approach to finding singular solutions of barotropic Euler's equations. The key idea is to find a constraint to be added to the original system making it overdetermined but compatible. This allows one to find multivalued solutions as integral manifolds of the Cartan distribution and study their singularities. Finding such constraints is based on the symmetry analysis of the equations in question and investigating the structure of the field of rational differential invariants of their symmetry group.

**Thursday 12th, 13:15 - 14:00**

**Roman Golovko** - On polyfillable Legendrian submanifolds in high dimensions
Abstract: Legendrian submanifolds of the standard contact $\mathbb{R}^{2n+1}$, $n$ $geq1$, which admit exact Lagrangian fillings are called fillable and belong to the rigid contact geometry. We will discuss the consequences of being fillable. After that, we will discuss Legendrian submanifolds which admit (at least one) infinite family of exact Lagrangian fillings that are diffeomorphic, but not Hamiltonian isotopic. Quite a few of the discussed Legendrian submanifolds will admit a high number of topologically distinct infinite families of diffeomorphic, but not Hamiltonian isotopic exact Lagrangian fillings.

**Jonatan Kogan** - Excision - from classical to $\infty$-categorical
The talk is expository, and will aim to explain the motivation of some $\infty$-categorical concepts and constructions from the perspective of classical algebraic-topology, specifically how the property of excision gives rise to the concept of an excisive functor and how this, in turn, gives the desired long exact sequences and other results.

**Wednesday 13th, 13:15 - 15:00**

**Christos-Raent Onti** - A class of Einstein submanifolds of Euclidean space
Abstract: The knowledge on the subject of Euclidean Einstein submanifolds, except those with constant sectional curvature, is quite limited. In fact, as far as we know, until now the only classification result available under purely intrinsic assumptions is in the case of hyper surfaces,

due to an observation by Cartan communicated by Thomas in 1937 and the work of Fialkow from 1938. In the talk, I will discuss the characterisation of a class of Einstein manifolds isometrically immersed into Euclidean space as rotational submanifolds. The highlight is for submanifolds in codimension two since in this case our assumptions are purely intrinsic. This is a joint work with Marcos Dajczer (IMPA) and Theodoros Vlachos (University of Ioannina)

**Itamar Mor** - Galois theory and Picard groups in homotopy theory
The Picard group classifying invertible objects is a fundamental invariant of symmetric monoidal (infinity-)categories. A problem first posed by Hopkins, Mahowald and Sadofsky is to determine these groups for the category of $K(n)$-local spectra, where $K(n)$ is a Morava K-theory. A very fruitful recent approach to this and related problems has been to use Galois theory: given a finite Galois extension $A \to B$ of ring spectra, one can try to obtain the Picard group of $A$ from that of $B$, at the expense of passing to the Picard space. Passing to a Galois extension can drastically simplify the problem, and the resulting homotopy fixed point spectral sequence is tractable in many cases of interest. In chromatic homotopy theory, one might hope to apply this technique to the (profinite, $K(n)$-local) Galois extension $L_K(n)S \to E_n$, since the $K(n)$-local Picard group of modules over the base is precisely the group considered by Hopkins-Mahowald-Sadofsky. As a first step in this direction, Heard, Mathew and Stojanoska used finite Galois descent to prove, at height $n = p - 1$ and for any intermediate extension $F$ over which $E_n$ is finite, that the Picard group of modules over $F$ is cyclic. I will introduce the formalism of Galois theory and Galois descent for commutative ring spectra, and explain their applications to Picard groups. I will talk briefly about the example above, and end by discussing some approaches (and obstacles) to extending this to the case of a profinite extension such as $L_K(n)S \to E_n$. The talk will be mostly expository.

**Andrs Angel** - Equivariant bordism in dimension 2 and counterexamples to the evenness conjecture of equivariant unitary bordism
Abstract: For a finite group, The equivariant bordism ring is a module over the usual (non-equivariant) cobordism ring. For $G$ abelian or metacyclic, the equivariant unitary bordism ring is a free module with generators in even degrees. It was conjecture that this should be true for general group, and then B.Uribe promoted the problem at ICM 2018, where he called it the evenness conjecture for equivariant bordism. We showed that the conjecture is false by finding explicit counterexamples and explicitly described the 2-dimensional equivariant unitary and oriented bordism groups for all finite groups. This talk is based on joint work with Eric Samperton, Carlos Segovia and Bernardo Uribe

**Rudradip Biswas** - Stable categories and complete cohomology for various classes of topological groups
Abstract: Stable module categories, in representation theory, are usually studied over group algebras of finite groups. In recent years, reevaluating the importance of an old paper by Benson, some developments have happened in constructing well-behaved stable module categories of certain infinite discrete groups. In this talk, I will present methods to take the treatment forward to a class of topological groups called TDLC groups (totally disconnected locally compact) satisfying certain cohomological finiteness properties. If time permits, I will comment on how using tools from Clausen-Scholze's "Condensed Mathematics", one can achieve similar objectives.

**Florian Schwarz** - Tangent infinity categories and why they are relevant
Abstract: In this talk I will sketch the definitions of tangent categories and tangent infinity categories. I will explain why these are interesting objects. By focusing on structural properties, tangent categories allow us to use concepts from differential geometry, like connections, vector bundles and flows without actually having a manifold. A tangent category has an axiomatic definition. By work of Leung, it can be equivalently described by a functor from the category of finitely generated Weil-algebras over the natural numbers. In order to use tangent categories in homotopy theory, one needs to formulate them up to homotopy, something best described by infinity categories. In order to do this, the characterization of tangent categories by Leung is used as it can be generalized easily to the infinity-category setting. Using Leung's characterization, one obtains a version of tangent infinity categories that can be applied to Goodwillie functor calculus.

**Rima Chatterjee** - A structure theorem of Legendrian knots in overtwisted manifolds
Abstract: Legendrian knots, their classification and structure theorems are an interesting area of study in contact 3- manifolds. Though there has been some work on Legendrian knots in tight contact manifolds, we know very little about these knots in overtwisted manifolds. This talk will focus on a structure theorem of Legendrian knots in overtwisted manifolds. I will talk about how knots behave under cabling in an overtwisted manifold and discuss the conditions when the geometric property of these knots is preserved under cabling. This is a joint work with John Etnyre, Hyunki Min and Anubhab Mukherjee.

## Thursday 14th, 11:30 - 12:30

**Wilson Forero** - The Glvez-Kock-Tonks conjecture for locally discrete decomposition spaces

Abstract: Glvez-Carrillo, Kock, and Tonks constructed a decomposition space $U$ of all Mbius intervals, as a recipient of Lawvere's interval construction for Mbius categories, and conjectured that $U$ enjoys a certain universal property: for every Mbius decomposition space $X$, the space of culf functors from $X$ to $U$ is contractible. In this talk, we work at the level of homotopy 1-types to prove the first case of the conjecture, namely for locally discrete decomposition spaces. This provides also the first substantial evidence for the general conjecture.

**Boris Botvinnik** - Families of diffeomorphisms and concordances detected by trivalent graphs
Abstract: (joint with T. Watanabe). We study families of diffeomorphisms detected by trivalent graphs via the Kontsevich classes. We specify some recent results and constructions by Watanabe to show that those non-trivial elements in homotopy group $\pi_*(BDiff_\partial(D^d)) \otimes \mathbb{Q}$ are lifted to homotopy groups of the moduli space of $h$-cobordisms $\pi_*(BDiff_\sqcup(D^d \times I)) \otimes \mathbb{Q}$. As a geometrical application, we show that those elements in $\pi_*(BDiff_\partial(D^d)) \otimes \mathbb{Q}$ for $d \geq 4$ are also lifted to the rational homotopy groups $\pi_*(\mathcal{M}_\partial^{psc}(D^d)_{h_0}) \otimes \mathbb{Q}$ of the moduli space of positive scalar curvature metrics.

**Thomas Mikhail** - A very brief introduction to Homotopy Type Theory
Abstract: Focusing on the identity type and the structure it endow types with, this talk offers a crash course in homotopy type theory.

**Vanny Doem** - Coloring Pseudo-Manifolds
I have been working on Coloring Pseudo-Manifolds. I showed that the chromatic number $X(G)$ of a $d$-dimensional Pseudo-Manifold $G$ is between $d+1$ and $2(d+1)$. I literally proved this theorem by using the concept from graph coloring and Cohomology. I also found many other related results in the join and product operations of graphs and solved the sphere coloring problems. Therefore, for this contributed talk, I am willing to present the participants with these findings.