Geometry and Topology, ICM sectional workshop

University of Copenhagen
6-14 July 2022

Titles and Abstracts

Thursday 7th

Bruno Klingler - Hodge theory, between algebraicity and transcendence
Abstract: The Hodge theory of complex algebraic varieties is at heart a transcendental comparison of two algebraic structures. We survey the recent advances bounding this transcendence, obtained mainly through the use of o-minimal geometry.

Kai Cieliebak - Lagrange multiplier functionals and their applications in symplectic geometry and string topology
Abstract: I will discuss the role of Lagrange multiplier functionals in mathematics and physics. The main focus is on Rabinowitz' action functional and its usage in symplectic geometry, as well as recent applications in string topology and the study of closed geodesics.

Kathryn Mann - Groups acting at infinity
Abstract: Studying rigidity of group actions by understanding behavior "at infinity" is an idea dating back to Selberg and Mostow. My recent work has exploited this perspective in the topological rather than algebraic setting to prove rigidity results for families of group actions that are basic examples in geometric topology: the actions of hyperbolic groups on their boundaries, actions of surface fundamental groups and mapping class groups on the circle, and actions built out of Anosov flows on 3-manifolds. My talk will survey recent rigidity and classification results and explain some of the unifying themes of this work.

Karen Vogtmann - Spaces of graphs

Abstract: Many phenomena in mathematics and science can be modeled by drawing finite metric graphs. The set of all such models forms a geometric space, which may be enhanced by giving the graphs additional structure. The topology of these spaces is still quite mysterious and is the focus of current activity in several different fields, including geometric group theory, low-dimensional topology, perturbative quantum field theory, number theory and tropical algebraic geometry. In this talk I will describe spaces of graphs in general, then focus on their applications to geometric group theory and point to some recent developments in this field.

Friday 8th

Jacob Rasmussen - Floer homology of 3-manifolds with torus boundary

Abstract: Manifolds with torus boundary have played a special role in the study of Floer homology for 3-manifolds since the earliest days of the subject. I will describe a simple geometric representation of the Floer homology of such a 3-manifold (defined jointly with Hanselman and Watson). I will explain a generalization of this theory to sutured manifolds and give applications to the topology of knots and 3-manifolds.

Alexander Kuznetsov - Homological algebraic geometry

Abstract: The idea of studying the geometry of an algebraic variety through the structure of its derived category of coherent sheaves goes back to the pioneering works of Bondal and Orlov on the verge of the millennium. One of the central concepts of this approach is that of a semiorthogonal decomposition. In my talk I will overview the (rapidly developing) story of semiorthogonal decompositions, touching on some of its most fascinating aspects: (1) Semiorthogonal components with interesting properties and their geometric significance; (2) Categorical extensions of classical geometric constructions (homological projective duality, categorical joins and cones, categorical resolutions of singularities); and (3) Completely new constructions such as categorical absorptions of singularities.

Michel van den Berg - Noncommutative crepant resolutions
Abstract: A crepant resolution- if existing- is in in some sense the best possible smooth approximation to a singular algebraic variety. Crepant resolutions admit natural non-commutative analogues. During the lecture I will discuss how these appear in various contexts.

Jennifer Hom - Homology cobordism and Heegaard Floer homology
Abstract: Under the operation of connected sum, the set of three-manifolds form a monoid. Modulo an equivalence relation called homology cobordism, this monoid (of homology spheres) becomes a group. What is the structure of this group? What families of three-manifolds generate, or don't generate, this group? We give some answers to these questions using Heegaard Floer homology. This is joint work with various subsets of I. Dai, K. Hendricks, M. Stoffregen, L. Truong, and I. Zemke.

Chi Li - Canonical Kaehler metrics and stability of algebraic varieties
Abstract: We survey some recent developments in the study of canonical Kähler metrics on algebraic varieties and their relation with stability in algebraic geometry.

Amnon Neeman - Finite approximations as a tool for studying triangulated categories
Abstract: A metric on a category assigns lengths to morphisms, with the triangle inequality holding. This notion goes back to a 1974 article by Lawvere. We'll begin with a quick review of some basic constructions, such as forming the Cauchy completion of a category with respect to a metric. And then will begin a string of surprising new results. It turns out that, in a triangulated category with a metric, there is a reasonable notion of Fourier series, and an approximable triangulated category can be thought of as a category where many objects are the limits of their Fourier expansions. And then come two types of theorems: (1) theorems providing examples, meaning showing that some category you might naturally want to look at is approximable, and (2) general structure theorems about approximable triangulated categories. And what makes it all interesting is (3) applications. These turn out to include the proof of a conjecture by Bondal and Van den Bergh, a major generalization of a theorem of Rouquier's, a short, sweet proof of Serre's GAGA theorem, and a proof of a conjecture by Antieau, Gepner and Helle

Saturday 9th

Thomas Nikolaus - Frobenius homomorphisms in higher algebra
Abstract: We will discuss instances of Frobenius homomorphisms in homotopy theory. Similar to ordinary algebra, these morphism play an important role in higher algebra and are related to various concepts in stable homotopy theory and algebraic K-theory. We also discuss the notion of perfectness, which is to say that these morphisms are equivalences, and relate this notion to the Segal conjecture, the red-shift conjecture and the classification of spaces by stable data. We also present new calculations of K- theory groups based on those ideas.

Richard Evan Schwartz - A Survey Lecture on Billiards
Abstract: I will give a survey lecture on billiards, concentrating on polygonal billiards, both rational and irrational, and also polygonal outer billiards. For some of the lecture I will focus on my own work, particularly the solution of the Moser-Neumann problem concerning the existence of unbounded orbits for outer billiards. I will illustrate my talk with colorful computer demos.

Danny Calegari - Sausages and butcher paper
Abstract: For each d > 1 the shift locus of degree d, denoted S_{d}, is the space of normalized degree d polynomials in one complex variable for which every critical point is in the attracting basin of infinity under iteration. It is a complex analytic manifold of complex dimension d - 1. We are able to give an explicit description of S_{d} as a complex of spaces over a contractible Ã_{d-2} building, and to describe the pieces in two quite different ways:
(1) (combinatorial): in terms of dynamical extended laminations; or
(2) (algebraic): in terms of certain explicit 'discriminant-like' affine algebraic varieties.
From this structure one may deduce numerous facts, including that S_{d} has the homotopy type of a CW complex of real dimension d - 1; and that S_{3} and S_{4} are K(π,1)s.
The method of proof is rather interesting in its own right. In fact, along the way we discover a new class of complex surfaces (they are complements of certain singular curves in C^2) which are homotopic to locally CAT(0) complexes; in particular they are K(π, 1)s.

Sunday 10th

Peter Hintz, Gustav Holzegel - Recent Progress in General Relativity
Abstract: We review recent progress in general relativity. After a brief introduction to some of the key analytical and geometric features of the Einstein equations, we focus on two main developments: the stability of black hole solutions, and the formation, structure, and dynamical stability of singularities.

Nathalie Wahl - Homological Stability: a tool for computations
Abstract: Homological stability has shown itself to be a powerful tool for the computation of the homology of families of groups such as general linear groups, mapping class groups or automorphisms of free groups. We give an overview of this subject, illustrated by examples.

Monday 11th

David Fisher - Totally geodesic submanifolds, invariant measures, and arithmeticity
Abstract: I will discuss recent work with Bader, Miller and Stover and emphasize connections between dynamics and geometry and topology. In particular, study real and complex hyperbolic manifolds with many totally geodesic submanifolds of dimension at least 2 and characterize them as being arithmetic. The proof begins by thinking of the object of study as an invariant measure for a group action instead of a submanifold and I'll motivate the results and explain the connections.

Tobias Hock Colding - Geometry of PDEs
Abstract: Optimal geometric structures and the evolution of shapes are governed by partial differential equations. These same types of equations come up over and over again across many diverse areas in science, engineering and mathematics.
The geometric invariance makes the equations canonical, and means that they also describe phenomena seemingly unrelated to geometry.
Often the geometry unlocks the structure of the equation and leads to fundamental tools in PDE. Conversely, analysis has played a central role

in the development of geometry. Understanding the equations and their fundamental properties requires simultaneous insight into both analysis and geometry and the interplay between the two. In this talk we will discuss this principle for several fundamental equations. We start by seeing how a long-standing problem in geometry leads to optimal regularity for viscosity solutions of a degenerate elliptic PDE, then turn to using PDE to understand optimal shapes and geometric evolution.

Robert J Young - Composing and decomposing surfaces in R^{n}
Abstract: How do you build a complicated surface? How can you decompose a surface into simple pieces? Understanding how to construct an object can help you understand how to break it down. We will present some constructions and decompositions of surfaces based on uniform rectifiability and use these decompositions to study problems in geometric measure theory and metric geometry.

Mark Mclean - Floer Cohomology and Birational Geometry
Abstract: We explain a few recent results applying Floer cohomology to topics in birational geometry. We show how one can prove part of the cohomological McKay correspondence by computing a Floer cohomology group in two different ways. After that we illustrate how Hamiltonian Floer cohomology can be used to prove that birational Calabi-Yau manifolds have the same small quantum cohomology algebras.

Tuesday 12th

Pierre-Emmanuel Caprace - An invitation to simple locally compact groups
Abstract: This talk is devoted to topological groups that are locally compact, non-discrete and simple. Prominent examples are provied by real simple Lie groups and simple algebraic groups over local fields. Further examples arise as groups acting on trees, on buildings or on CAT(0) cube complexes. The aim of the talk is to explain how that class of simple groups as a whole has recently become an independent subject of study. This talk supplements the ICM talk, that will be presented online jointly with George Willis according to the ICM schedule. The two talks are independent.

Krishnamurthi Ramasubramanian - The History and Historiography of the Discovery of Calculus in India

Abstract: Couched in rich poetic verses in the Sanskrit language, the history of mathematics in India provides a fertile field for researching the evolution of mathematical thinking. During the talk, starting with snippets from the work of Āryabhaṭa (c. 499 CE) we shall try to present how certain important breakthroughs lead to the pioneering contribution of Mādhava (c. 1340) of the Kerala School, which has a more direct bearing on calculus. Towards the end, we would also like to highlight some of interesting facets in the historiography pertaining to development of calculus in India.

Oscar Randal-Williams - Diffeomorphisms of discs

Abstract: I will describe what is currently known, for $d \geq 5$, about the rational homotopy type of the group of diffeomorphisms of the d-disc relative to its boundary, and the closely related group of homeomorphisms of d-dimensional Euclidean space.

Wednesday 13th

Richard Bamler - Some recent developments in Ricci flow

Abstract: Ricci flows are a powerful geometric-analytical tool, as they have been used to prove important results in low-dimensional topology. In the first part of this talk I will focus on Ricci flows in dimension 3. I will briefly review Perelman's construction of Ricci flow with surgery, which led to the resolution of the Poincare' and Geometrization Conjectures. Then I will discuss recent work of Lott, Kleiner and myself on an improved version of this flow, called "singular Ricci flow". This work allowed us to resolve the Generalized Smale Conjecture, concerning the topology of diffeomorphism groups, and a conjecture concerning the contractibility of the space of positive scalar curvature metrics on 3-manifolds. In the second part of the talk, I will focus on Ricci flows in higher dimensions. I will present a new compactness theory, which can be used to study the singularity formation of the flow, as well as its long-time asymptotics. I will discuss these and some further consequences. I will also convey some intuition of the new terminology that had to be introduced in connection with this compactness theory.