Holomorphic Day 2023

Program, abstracts and practical information

The 17th of November, 2023

University of Copenhagen, Copenhagen, Denmark

The Conference is sponsored by the Danish Council for Independent Research | Natural Sciences via grant DFF – 1026-00267B.

Practical information

The meeting is scheduled for Friday the 17th of November at the Department of Mathematical Sciences of the University of Copenhagen

Department of Mathematical Sciences
Universitetsparken 5
Auditorium 10
2100 Copenhagen Ø

Auditorium 10 is equipped with blackboards and a beamer.

After the talks we have organized a common dinner for those that have registered. More information will given during the day.

You may access the eduroam or KU-guest networks.

Program

09:30 – 10:00	Coffee in the lunch room the the fourth floor of the math building
10:00 – 10:50	Joaquim Ortega-Cerdà: Hypercontractive inequalities of complex polynomials
11:00 – 11:50	Tomoki Kawahira: Visualization of quasiconformal deformations of holomorphic dynamics
11:50 – 13:30	Lunch
13:30 – 14:20	Yacin Ameur: The partition function for Coulomb gas ensembles with spectral gaps
14:20 – 15:00	Coffee
15:00 – 15:50	Turgay Bayraktar: Dynamics of asymptotically minimal polynomials
16:00 – 16:50	Carsten Lunde Petersen: Taming the elephants in the Mandelbrot set
17:30 –	Common dinner for registered participants

Abstracts

The partition function for Coulomb gas ensembles with spectral gaps

YACIN AMEUR

LUND UNIVERSITY, SWEDEN

Consider a plasma consisting of n repelling point charges $\{z_j\}_1^n$ in the complex plane $\mathbb{C}$, subjected to a suitable confining potential Q, which is "large" near infinity and radially symmetric. The particles will tend to occupy a droplet con- sisting of concentric annuli, possibly with a central disk. There might also be some "spectral outposts", i.e., components of the coincidence set outside of the droplet. The energy of the system is taken to be

$$H_n = \sum_{j \neq k} \log \frac{1}{|z_j - z_k|} + n \sum_{j=1}^{n} Q(z_j).$$

We consider a large n expansion for the free energy $\log Z_n$ (at inverse temperature $\beta = 2$), where $Z_n = \int_{\mathbb{C}^n} e^{-H_n}$ is the partition function. The expansion takes the form

$$\log Z_n = C_1 n^2 + C_2 n \log n + C_3 n + C_4 \log n + C_5 + \mathcal{G}_n + o(1),$$

where $C_1, \ldots, C_5$ are certain geometric functionals while the oscillatory term $\mathcal{G}_n$ is expressed in terms of Jacobi's theta function. The coefficients C_4, C_5 and $\mathcal{G}_n$ are related to the topology of the droplet and to fluctuations of linear statistics; in particular $\mathcal{G}_n$ is interpreted as a sum of Heine distributed random variables which measure "jumps" from a given component of the coincidence set to another one. Joint work with Joakim Cronvall and Christophe Charlier.

Dynamics of asymptotically minimal polynomials

TURGAY BAYRAKTAR

FACULTY OF ENGINEERING AND NATURAL SCIENCES, SABANCI UNIVERSITY, TURKEY

In this talk, we will focus on dynamical properties of asymptotically extremal polynomials (in the sense of Stahl and Totik) associated with a non-polar planer compact set. In particular, we shall prove that if the zeros of such polynomials are uniformly bounded then their Brolin measures converge weakly to the equilibrium measure of the compact set. This is joint work with M. Efe.

Visualization of quasiconformal deformations of holomorphic dynamics

TOMOKI KAWAHIRA

HITOTSUBASHI UNIVERSITY, JAPAN

Structural stability of holomorphic dynamics (iteration of polynomial/rational maps, Fuchsian/Kleinian groups, etc.) on the Riemann sphere can be characterized by quasiconformal deformability. More precisely, a (holomorphic) family of holomorphic dynamics is structurally stable if each dynamics of this family is conjugate to the others by quasiconformal mappings. We usually recognize this fact only theoretically, and we do not see how the actual quasiconformal conjugations look like. In this talk, I will present some attempts to visualize such quasiconformal mappings by solving the Beltrami equations numerically. The algorithm I use is a variant of the Porter-Shimauchi method, which seems very efficient for our purpose.

Hypercontractive inequalities of complex polynomials

JOAQUIM ORTEGA-CERDÀ

UNIVERSITY OF BARCELONA, SPAIN

If we endow the polynomials in one complex variable with the Bombieri norm, we have a Hilbert space of holomorphic functions with reproducing kernel. We will show that every convex functional of the pointwise norm of the normalized functions in the space achieves its extreme at the normalized reproducing kernels. This allows us to give a new, very elementary proof, of a physical conjecture about the behavior of the Wehrl entropy for Bloch coherent states, which was initially proved by Lieb and Solowej. We will also obtain results analogous to the Faber-Krahn inequality in the context of polynomials. This is a joint work with N. Fabio, A. Kulikov and P. Tilli. If time allows it, we will present some more recent results regarding the stability of such type of inequalities.

Taming the elephants in the Mandelbrot set

CARSTEN LUNDE PETERSEN

ROSKILDE UNIVERSITY, DENMARK

We study classes of transcendental Horn and Adam maps, which contain the classical horn maps and their quotients coming from the dynamics on simple parabolic basins such as $z^2 + 1/4$ on the Cauliflower. We define and describe the structure of the dynamical and parameter "lagoa rays" and use these rays to design the Yoccoz puzzle machinery in this setting. As a first consequence, we obtain the precise asymptotics of secondary satellite copies of "principal elephants" near the cusp of the Mandelbrot set.