

Holomorphic Day 2022

Program, abstracts and practical information

November 18, 2022

University of Copenhagen, Copenhagen, Denmark

Version of November 16, 2022

The Conference is sponsored by the Danish Council for Independent Research | Natural Sciences via grant DFF – 1026-00267B.

Practical information

The meeting is held Friday November 18 at the Department of Mathematical Sciences of the University of Copenhagen

Department of Mathematical Sciences
Universitetsparken 5
Auditorium 10
2100 Copenhagen Ø

Auditorium 10 is equipped with blackboards and a beamer. The coffee breaks take place outside the auditorium. We plan a common dinner in the evening. More information will be given during the day.

You may access the eduroam or KU-guest networks.

Program

09:30 – 10:00	Coffee
10:00 – 10:50	Meng Yang Strong Asymptotics of Planar Orthogonal Polynomials: Gaussian Weight Perturbed by Point Charges
11:00 – 11:50	Annemarie Luger Herglotz-Nevanlinna functions in several variables
11:50 – 13:30	Lunch
13:30 – 14:20	Mikhail Sodin Translation-invariant probability measures on entire functions
14:20 – 15:00	Coffee
15:00 – 15:50	Weiwei Cui Perturbations of non-recurrent dynamics in the exponential family
16:00 – 16:50	Signe Emalia Jensen Geometric limits of Julia sets for uncritical polynomials under degree growth
17:30 –	Common dinner

Abstracts

Perturbations of non-recurrent dynamics in the exponential family

WEIWEI CUI

LUND UNIVERSITY, SWEDEN

We study maps from the exponential family

$$\{\lambda e^z \mid \lambda \in \mathbb{C} \setminus \{0\}\}$$

for which the singular value is non-recurrent. We show that the set of non-recurrent maps in the parameter space has Lebesgue measure zero. Moreover, one can approximate non-recurrent parameters by hyperbolic ones. This is a joint work with Magnus Aspenberg.

Geometric limits of Julia sets for uncritical polynomials under degree growth

SIGNE EMALIA JENSEN

Consider the family $f_{n,c}(z) = z^n + c$ of unicritical polynomials of degree $n \geq 2$. Each fixed complex number c has an associated sequence of unicritical polynomials $f_{n,c}$ as well as a sequence of Julia sets $J(f_{n,c})$. This talk is about the limit of the geometric sets $J(f_{n,c})$ as the degree n tends to infinity. The main question is: for which c does the associated sequence of Julia sets have a geometric limit? The geometry of a Julia set $J(f_{n,c})$ depends on the location of c in the parameter plane of degree n , so the study of geometric limit sets is simultaneously a study of how the connectedness locus changes as the degree increases.

Herglotz-Nevanlinna functions in several variables

ANNEMARIE LUGER

STOCKHOLM UNIVERSITY, SWEDEN

A holomorphic function $f : (\mathbb{C}^+)^n \rightarrow \mathbb{C}$ is called *Herglotz-Nevanlinna* if it satisfies $\text{Im}(f(\vec{z})) \geq 0$ for all points \vec{z} in the poly upper half plane $(\mathbb{C}^+)^n$.

For $n = 1$ these functions are known also as Pick functions or R-functions, and they are very well studied. In particular, it is known for about 100 years that the class of these functions is (via an integral representation) in a one-to-one correspondence to the set of data triples (a, b, μ) , where $a \in \mathbb{R}$, $b \geq 0$, and μ is a Borel measure satisfying a natural growth condition.

For $n \geq 2$ the situation is quite different. Recent results show that these functions are still given via a similar integral representation, however, not all reasonable measures are admissible, and there is no constructive description of the class of admissible measures.

We will give an overview on what is known about these functions in several variables, and point out questions that are still open.

This talk is based on joint work with Mitja Nedic.

Translation-invariant probability measures on entire functions

MIKHAIL SODIN

TEL AVIV UNIVERSITY, ISRAEL

I shall speak about a somewhat unexpected object: the probability measures on the space of entire functions (of one complex variable) which are (a) invariant with respect to the action of the complex plane by translations, and (b) do not charge the constant functions. The existence (and even an abundance) of such measures was discovered by Benjy Weiss.

My talk will be mostly based on a joint work with Lev Buhovsky, Adi Glucksam, and Alexander Logunov <https://arxiv.org/abs/1703.08101>

Strong Asymptotics of Planar Orthogonal Polynomials: Gaussian Weight Perturbed by Point Charges

MENG YANG

UNIVERSITY OF COPENHAGEN, DENMARK

We consider the planar orthogonal polynomials $\{p_n(z)\}$ with respect to the measure supported on the complex plane

$$e^{-N|z|^2} \prod_{j=1}^{\nu} |z - a_j|^{2c_j} dA(z)$$

where dA is the Lebesgue measure of the plane, N is a positive constant, $\{c_1, \dots, c_\nu\}$ are nonzero real numbers greater than -1 and $\{a_1, \dots, a_\nu\} \subset \mathbb{D} \setminus \{0\}$ are distinct points inside the unit disk. When $\nu = 1$, in the scaling limit $n/N = 1$ and $n \rightarrow \infty$, we obtain strong asymptotics of $p_n(z)$ via a matrix Riemann–Hilbert problem. From the asymptotic behavior of $p_n(z)$, we find that, as we vary c_1 , the limiting distribution of zeros behaves discontinuously at $c_1 = 0$. We also derive the strong asymptotics of $p_n(z)$ for the case of $\nu > 1$ by applying the nonlinear steepest descent method on the matrix Riemann-Hilbert problem of size $(\nu + 1) \times (\nu + 1)$. This talk is based on joint work with Seung-Yeop Lee.