Masterclass on Higher Representations: Abstracts

Centre for Geometry & Topology, University of Copenhagen

Week 1

David Treumann

Five lectures on algebraic topology in modular representation theory.

I will discuss two themes (1. Smith theory and 2. G-spectra) connecting algebraic topology to modular representation theory. "Smith theory" is a relation between topology of X and topology of the fixed points of a $\mathbb{Z}/p$-action on X. It has some compelling applications to mod p automorphic forms and (via geometric Satake) to the representation theory of mod p algebraic groups. "G-spectra" are spectra equipped with G-actions; the philosophy from the 1900s is that they are natural generalizations of abelian groups equipped with G-actions. I wish I could present a compelling application of this philosophy in representation theory. Instead, I will argue that it raises some compelling questions in algebraic topology, and present answers to those questions in simple cases. And I will tell some speculations about applications going the other way.

Some talks may present joint work of myself and Venkatesh, and of myself and Feng and Yuan. Some talks may present work of others e.g. Leslie and Lonergan, Riche and Williamson.

Lennart Meier

Lecture 1: Equivariant K-theory

Equivariant K-theory serves as the blueprint for equivariant elliptic cohomology. We will recall the definition of equivariant K-theory, discuss its role in representation theory and sketch an alternative construction of equivariant K-theory by inducing up from abelian subgroups.

Lecture 2: Introduction to equivariant elliptic cohomology

We will motivate the notion of equivariant elliptic cohomology and present a construction. This will include a brief discussion of spectral algebraic geometry.

Lecture 3: Properties of equivariant elliptic cohomology and outlook

We will discuss some basic computations regarding elliptic cohomology and present an outlook, connecting in particular to other "higher variants" of equivariant K-theory.

Tobias Barthel

Title: Quillen stratification in higher representation theory.

Abstract: In this talk we will discuss various generalizations of Quillen's stratification theorem, with a particular focus on its incarnations in equivariant tensor-triangular geometry. Our goal is to explain the computation of the Balmer spectrum as well as the cohomological parametrization of all localizing tensor-ideals of the category of equivariant modules over Borel-equivariant Lubin–Tate theory, thereby establishing a mixed chromatic characteristic analogue of the work of Benson, Iyengar, and Krause in modular representation theory. This is joint work with Natalia Castellana, Drew Heard, Niko Naumann, and Luca Pol.

Week 2

Akhil Mathew

Title: **K-theory and chromatic homotopy theory**

The interactions between chromatic homotopy theory and algebraic K-theory, especially around the theme of chromatic redshift proposed by Ausoni and Rognes, have seen a number of remarkable advances in recent years. I will explain some work with Clausen, Naumann, and Noel and Land, Meier, and Tamme on the Galois descent and purity properties of telescopically localized K-theory. The Galois descent results rely on the use of the study of K-theory of representation categories of p-groups and the use of vanishing of Tate constructions in telescopic homotopy theory.

Robert Burklund

Title: **Spectral representations of C_p**

Abstract: In this series of 3 lectures I will introduce the subject of spectral representations of finite groups, focusing mostly on the case of the cyclic group of prime order C_p. A spectral representation of a finite group G is a finite spectrum equipped with a G-action, or in other terms, a representation of G with coefficients in the sphere spectrum. The main focus of the lectures will be determining the spectral representation ring of C_p. Along the way we will encounter a new perspective on group actions on stable spheres and a categorification of the Segal conjecture for C_p. This is joint work with Jesper Grodal and Allen Yuan.

Lior Yanovski

Title: **Higher semiadditive algebra and representation theory.**

Abstract: In stable homotopy theory, the Morava K-theory ring spectra $K(n)$ play the role of prime fields in classical algebra, and in a sense interpolate between characteristic 0 (at height $n = 0$) and characteristic p (at height $n = \infty$). The ∞-categories of $K(n)$-local spectra for finite n possess a remarkable property called higher semiadditivity (which was subsequently shown to hold for the telescopic localizations as well). Roughly speaking, it is the property that gives a canonical way to integrate families of morphisms indexed

by π-finite spaces, generalizing the ordinary sum of morphisms in a semi-additive category. Moreover, the chromatic height n itself can be expressed in terms of this higher semiadditive structure and turns out to be one of its fundamental invariants. As is well known, the representation theory of finite groups behaves very differently in characteristic 0 and in characteristic p (e.g. in semisimplicity vs. unipotence results). In these talks, after reviewing some background material, I will explain how using higher semiadditive methods, one can provide height n analogs of some fundamental algebraic constructions that classically work only in characteristic zero, such as (primitive) roots of unity, cyclotomic extensions, and the discrete Fourier transform. Furthermore, I will discuss how these in turn can be used to study the Galois and representation theory of finite, and more generally π-finite, groups in the "intermediate characteristics". If time permits, I will also mention work in progress on applications to chromatically localized algebraic K-theory.