Dilation and Classification in Operator Algebra Theory - Titles and Abstracts

Mini-courses

Dilation Theory and Applications - Orr Shalit

Dilation theory is collection of results, tools and techniques in operator algebra theory that all stem from one big idea: you can learn a lot about an operator (or a map) by viewing it as a "part of" of another, well understood operator (or map). The manifestations of the idea being so many, and the applications so numerous, that there is no unique canonical way to arrange everything that falls under the title "dilation" into a single coherent and useful "theory". That does not mean that there isn't a whole lot that can be said.

The goal of this mini-course is to propose a point of view of what dilation theory is, and to tell the story of how dilation theory evolved during the last 70 years, how it is used, and where it is going. I hope that participants will acquire useful tools, learn about inspiring applications, and understand the broad context.

The first part of the lecture-series will be devoted to what I see as the core of the subject. These are the foundations that were laid in the 20th century:

1. Overview of Sz.-Nagy and Foias's single operator dilation theory and some applications.
2. Dilation theory for CP maps and the Stinespring-Arveson operator algebraic approach to dilation theory.
3. Multivariable generalizations and applications to operator algebras.

The second part will cover some 21st century versions of dilation theory and their applications. The topics covered and the depth that we shall go into will depend on the participant interest and the dynamics in the lectures, and will be chosen from the following:

1. The noncommutative Choquet boundary theory and the existence of C*-envelope.
2. Matrix convexity, minimal and maximal matrix convex sets, classification of operator systems.
3. CP-semigroups and E-dilations.
4. Exotic dilations and dilation constants, dilating noncommuting to commuting, dilations of q-commuting unitaries, applications.

Many references will be mentioned as we go and indicated in the slides. We shall use the following survey paper as a central reference:

[1] O.M. Shalit *Dilation theory: a guided tour*, Proceedings of IWOTA 2019, Lisbon, **Oper. Theory Adv. Appl.**, Vol. 282 Birkhäuser, 551–623, 2021. `https://arxiv.org/pdf/2002.05596v2.pdf`

Operator algebras and groupoids arising from left regular representations - Xin Li

This mini-course is about groupoids and C*-algebras generated by left regular representations of certain left cancellative small categories called Garside categories. I will explain what these objects are and illustrate our constructions with the help of many concrete example classes. I will also introduce boundary quotients and discuss how they build a connection to non-self-adjoint operator algebras. The main motivation to study Garside categories in our context comes from topological full groups associated with groupoids, which build a bridge to group theory by producing new examples of groups with very interesting properties, solving outstanding open problems in group theory. I will discuss important invariants and give an overview of structural results for C*-algebras, groupoids and topological full groups arising from Garside categories.

Mighty oaks from tiny acorns: An introduction to graph C^*-algebras and Cuntz-Pimsner algebras - Mark Tomforde

In the past few years striking progress has been made in the classification of graph C^*-algebras. This work has benefited from remarkable interactions with symbolic dynamics, groupoids, and K-theory, as well as the many visual tools provided by the graph itself. Currently, a variety of results for graph C^*-algebras are guiding researchers as they attempt to classify additional non-simple C*-algebras as well as understand the structure of more general types of C^*-algebras. One important (and vast) generalization of graph C^*-algebras are the Cuntz-Pimsner algebras, which are C^*-algebras constructed from certain Hilbert C^*-modules. Cuntz-Pimsner algebras were introduced in 1997 and studied extensively in the early 2000s, and in the past few years the subject has experienced a renaissance as researchers have found new applications and useful connections with other topics related to operator algebras.

This mini-course serves as an introduction to graph C^*-algebras and Cuntz-Pimsner algebras. We first explore the basic structure of graph C^*-algebras, with an emphasis on how the graph provides a tool for visualizing properties of the associated C^*-algebra. We then introduce Cuntz-Pimsner algebras, using the graph C^*-algebras as motivating examples. We shall see that, despite the fact Cuntz-Pimsner algebras are vastly more general, an understanding of graph C^*-algebras provides a useful mental framework for investigating the Cuntz-Pimsner algebras. After discussing current trends and methods in these subjects, we conclude with explanations of open problems for both graph C^*-algebras and Cuntz-Pimsner algebras that are currently being investigated by the research community.

Research Lectures

Reconstruction of twisted Steinberg algebras - Becky Armstrong

Steinberg algebras are a purely algebraic analogue of groupoid C*-algebras that generalise both Leavitt path algebras and Kumjian–Pask algebras. Since their introduction in 2010, Steinberg algebras have served as a bridge to facilitate the transfer of various concepts and techniques between the algebraic and analytic settings. Given the importance of *twisted* groupoid C*-algebras in C*-algebraic research (for instance, Renault's theorem that every C*-algebra with a Cartan subalgebra is a twisted groupoid C*-algebra), a natural question to ask is whether Steinberg algebras can be twisted in an analogous way. In this talk I will introduce twisted Steinberg algebras and various notions of *algebraic Cartan pairs* and will explain the important role these play in algebraic reconstruction theorems. (This is joint work with Gilles G. de Castro, Lisa Orloff Clark, Kristin Courtney, Ying-Fen Lin, Kathryn McCormick, Jacqui Ramagge, Aidan Sims, and Benjamin Steinberg.)

Algebraic semigroup actions, C*-algebras, and rigidity - Chris Bruce

Each algebraic semigroup action gives rise to an étale groupoid, and properties of the action translate into properties of the grouped C*-algebras. For instance, under mild assumptions on the action, the resulting reduced groupoid C*-algebra is simple and purely infinite. I will explain this construction and then discuss rigidity results for Cartan pairs in this setting. This is joint work with Xin Li.

Minimal boundaries for operator algebras - Raphaël Clouâtre

One spectacular achievement of modern dilation theory is the construction, for any operator algebra, of sufficiently many boundary representations. These representations are the driving force behind an ambitious program aiming to clarify the structure of non self-adjoint operator algebras through the use of non-commutative function theoretic methods. Indeed, the collection B of boundary representations can be meaningfully interpreted as a non-commutative analogue of the Choquet boundary of a function algebra. Guided by classical intuition, one can thus think of the closure of B as the minimal *closed* boundary of an operator algebra. In this talk, we will explore the general question of minimality for boundaries of operator algebras. We will relate this question to non-commutative notions of peak points and to what we call the Bishop property.

Wold decomposition and C*-envelope from self-similar graphs - Boyu Li

A self-similar graph encodes two-way actions between a semigroup and a directed graph. We first derived a Wold-type decomposition for its Nica-covariant representations, breaking its representations into a direct sum of four components. One component gives the boundary representation, and we can dilate the other three into the boundary representation. As a result, the C*-envelope is generated by the boundary representations. This is joint work with Dilian Yang.